Three problems on pancyclic edges

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A k-cycle is a cycle of length k.

An edge e of a graph of order n is said to be *pancyclic* if for every integer k with $3 \leq k \leq n$, e lies in a k-cycle.

If $n \geq 3$ is an odd integer, we denote by $BT(n)$ the graph obtained by identifying an edge of $K_{(n-1)/2,(n-1)/2}$ with an edge of C_3.

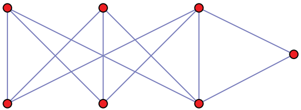

图: The graph $BT(7)$

Conjecture 1 (X. Zhan, July 2025). If G is a nonbipartite hamiltonian graph of order $n \geq 7$ with size at least $\lfloor (n-1)^2/4 \rfloor + 2$ other than $BT(n)$ when n is odd, then G contains a pancyclic edge.

A graph G of order n is said to be *vertex-pancyclic* if for every vertex v of G and for every integer k with $3 \leq k \leq n$, v lies in a k-cycle.

There exist vertex-pancyclic graphs that contain no pancyclic edge.

Problem 2 (X. Zhan, October 2025). Characterize the vertex-pancyclic graphs that contain no pancyclic edge.

The following question is a subproblem of Problem 2.

Question 3 (X. Zhan, October 2025). Does there exist a positive integer k such that every k-connected vertex-pancyclic graph contains a pancyclic edge?

- R. Häggkvist, R.J. Faudree and R.H. Schelp, Pancyclic graphs–connected Ramsey number, Ars Combin., 11(1981), 37–49.
- C. Li and X. Zhan, Every 2-connected $[4, 2]$-graph of order at least seven contains a pancyclic edge, arXiv:2511.07758, 11 November 2025.

T H A N K   Y O U