

# Slope inequalities and Miyaoka-Yau type inequality in positive characteristics

## Abstract:

The celebrated Miyaoka-Yau inequality  $c_1^2 \leq 3c_2$  (or equivalently  $c_1^2 \leq 9\chi$ ) for complex algebraic surfaces of general type is among the most useful results in algebraic geometry. In this talk, we shall study its positive characteristic analogue. We will start by reviewing the early works of Raynaud and Shepherd-Barron. Then we will give a positive characteristic version of Xiao's slope inequality. Finally, with the help of this slope inequality, we will present the Miyaoka-Yau type inequality for algebraic surface of general type in positive characteristic  $p > 0$  as below:

- if  $p \geq 5$ , then  $c_1^2 \leq \frac{4(3p+1)(p-3)}{p^2-4p-1} \chi$ .
- if  $p = 3$ , then  $c_1^2 < 32\chi$ .
- if  $p = 2$ , then  $c_1^2 \leq 32\chi$ .

The inequalities are optimum for  $p = 2$  and  $p \geq 5$ . As a consequence of the inequality, we have  $\chi > 0$  holds true, answering a question of Shepherd-Barron completely.

This is a joint work with Professor Sun Xiaotao and Zhou Mingshuo.