Course: Commutative Algebra

Homework 9 (due to next Friday, 4/27/2012)

All rings are assumed commutative with identity.

1. Let k be an algebraically closed field and let $V = \mathcal{Z}(xz - yw) \subseteq \mathbb{A}^4$ and $f = \bar{x}/\bar{y}$. Prove that there is no single expression $f = a/b \in K(V)$ with $b(v) \neq 0$ for every v where f is regular.

2. Define the support of an R-module M by

$$\text{Supp}M = \{P \in \text{Spec}R | M_P \neq 0\}.$$

Prove the following results:

(a) $M = 0$ if and only if $\text{Supp}M = \emptyset$.

(b) If

$$0 \to L \to M \to N \to 0$$

is an exact sequence of R-modules then the localization M_P is nonzero if and only if L_P or N_P is nonzero and deduce that $\text{Supp}M = \text{Supp}L \cup \text{Supp}N$.

(c) Suppose $P \subseteq Q$ are prime ideals in R. Then the localization of the R-module M_Q at P is the localization M_P, i.e., $(M_Q)_P = M_P$. If $P \in \text{Supp}M$ then $Q \in \text{Supp}M$.