Course: Commutative Algebra

Homework 6 (due to next Friday, 3/30/2012)

All rings are assumed commutative with identity. k is a field.

1. Show that $(y^2 - x^3 - x^2) \lhd k[x,y]$ is prime ideal and find the integral closure of
$$A = k[x,y]/(y^2 - x^3 - x^2).$$

2. Suppose that S is integral over R and that P is a prime ideal in R. Prove that every element in the ideal PS generated by P in S is integral over P.

3. Let $f \in \mathbb{C}[x_1, \cdots x_n]$ be a square-free nonconstant polynomial. Let
$$A = \mathbb{C}[x_1, \cdots x_n, z]/(z^2 - f).$$
Show that A is integrally closed.

4. Let $R \subseteq S$ be rings. Show that R is integrally close in S if and only if $R[x]$ is integrally closed in $S[x]$.