Course: Commutative Algebra

Homework 3 (due to next Friday, 3/9/2012)

All rings are assumed commutative with identity.

1. Let I and J be ideals in the ring R. Prove the following statements:
   - $\text{rad}(IJ) = \text{rad}(I \bigcap J) = \text{rad}I \bigcap \text{rad}J$
   - $\text{rad}(\text{rad}I) = \text{rad}I$
   - $\text{rad}I + \text{rad}J \subseteq \text{rad}(I + J)$, $\text{rad}(I + J) = \text{rad}(\text{rad}I + \text{rad}J)$
2. If N is the nilradical of R, prove that $\text{Spec}R$ and $\text{Spec}R/N$ are homeomorphic with respect to Zariski topology.
3. Suppose R and S are rings and $\varphi : R \to S$ is a ring homomorphism. If I is an ideal of R show that $\varphi(\text{rad}I) \subseteq \text{rad}(\varphi(I))$. If in addition φ is surjective and I contains the kernel of φ show that $\varphi(\text{rad}I) = \text{rad}(\varphi(I))$.
4. Give an example of an injective k-algebra homomorphism $\tilde{\varphi} : A(W) \to A(V)$ whose associated morphism $\varphi : V \to W$ is not surjective.