Course: Commutative Algebra

Homework 13 (due to next Friday, 5/25/2012)

R is a commutative ring with identity.

1. Suppose R is an integral domain with fraction field K and P is a nonzero prime ideal in R. Show that the fraction ideals of R_P in K are the R_P modules of the form AR_p where A is a fractional ideal of R.

2. Suppose R is an integral domain with fraction field K and A is a fractional ideal of R in K. Let $A' = (R : A) = \{x \in K | xA \subseteq R\}$. Show that for any prime ideal P in R, $(A')_P$ is a fractional ideal of R_P in K. If A is a finitely generated R-module, then $(A')_P = (A_P)'$ where $(A_P)' = (R_P : A_P) = \{x \in K | xA_P \subseteq R_P\}$.

3. Suppose R is a Noetherian integral domain that is not a field. Prove that R is a Dedekind Domain if and only if for every maximal ideal M od R there are no ideal I of R with $M^2 \subsetneqq I \subsetneqq M$.

4. Suppose R is a Noetherian integral domain with Krull dimension 1. Prove that every nonzero ideal I in R can be written uniquely as a product of primary ideals whose radicals are all distinct.

5. If I and J are nonzero fractional ideals for the Dedekind Domain R with fraction field K. Prove that there are elements $a, b \in K$ such that aI and bJ are nonzero integral ideals in R and relatively prime.