Course: Commutative Algebra

Homework 12 (due to next Friday, 5/18/2012)

R is a commutative ring with identity.

1. Suppose R to be a D.V.R. with respect to the valuation ν on the fraction field K of R. If $x, y \in K$ with $\nu(x) < \nu(y)$, prove that $\nu(x+y) = \min(\nu(x), \nu(y))$.

2. Show that the ring of formal Laurent series $F((x))$ with coefficients in the field F is a field and has discrete valuation

   $$\nu : F((x))^* \to \mathbb{Z}$$

   by

   $$\nu(\sum_{n \geq N}^{\infty} a_n x^n) = N,$$

   where $a_N \neq 0$. So the corresponding D.V.R. is the ring of formal power series $F[[x]]$.

3. Suppose R is a Noetherian local ring whose unique maximal ideal $M = (t)$ is principal. Prove that either R is a D.V.R or $t^n = 0$ for some $n \geq 0$. In the latter case show that R is Artinian.