HINTS AND CONCLUSIONS FOR "3X"

All of you guys did a good job on your final, especially on No. 3, 5, 7. For No. 4, only Xu gave a correct answer. No. 8 is relatively hard. Xie's answer is right but Xu and Xing's answers are partially correct.

1. (2) If Y is projective, suppose L is any line bundle on Y. Then $f^*L \equiv \sigma^*\mathcal{O}_{\mathbb{P}^2}(b) + \sum a_i E_i$, where E_i is the exceptional curve above p_i, "$\equiv$"is linearly equivalence. $f^*L|_{\tilde{C}} \equiv 0$, so $\mathcal{O}_C(b) + \sum a_i p_i \equiv 0$ on C, which is impossible for general choice of p_i.

2. $\mu^*D = \tilde{D} + 2E_1 + 3E_2 + 6E_3$, $K_{X'/X} = E_1 + 2E_2 + 4E_3$.

4. Use Hodge Index Theorem to show two components. Notice that the self-intersection of any curve on abelian surface is non-negative.

6. The pencil $\lambda z(xy - z^2) + \mu(x-y)^3$ has singular fiber at $[\lambda : \mu] = [1 : 0], [0, 1], [36 : \sqrt{3}i], [-36 : \sqrt{3}i]$.

Figure of Y: