

## Harmonic Analysis: Home Work (Week 3)

2008年3月5日

♠ Exercises on the space of test function space  $S$

Prove the following statements:

- (1)  $S$  is complete under its topology;
- (2) The mapping  $\varphi \mapsto x^\alpha D^\beta \varphi$  is continuous;
- (3) The sequence  $\tau_h \varphi$  converges to  $\varphi$  as  $h \rightarrow 0$ ;
- (4) The sequence  $(\varphi - \tau_h \varphi)/h_j$  converges to  $\partial_j \varphi$  as  $h_j \rightarrow 0$  for  $\varphi \in S$ . Here  $h = (0, \dots, 0, h_j, 0, \dots, 0)$ ;
- (5)  $S$  is separable;
- (6) The Fourier transform is a homeomorphism on  $S$ ;
- (7)  $C_0^\infty(\mathbb{R}^n)$  is a dense subset of  $S$ ;
- (8)  $S \subset L^p$ ,  $S \subset C_0$  for  $1 \leq p \leq \infty$ .

♠ P. 42 of the text book, (21)

♠ Write out a short summary of the results, techniques, ideas (or philosophies, strategies) you have learned so far in this class.

Note: All the home works of week 1–week 3 should be submitted on April 9.