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Stable equivalences of Morita type and stable Hochschild cohomology rings

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Abstract. We prove that a stable equivalence of Morita type between finite dimensional algebras preserves the stable Hochschild cohomology rings, that is, Hochschild cohomology rings modulo the projective center, thus generalizing the results of Pogorzały and Xi.

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1. Introduction and the main result. Stable equivalences of Morita type were introduced by Broué [1]. This notion is closed related to that of derived equivalence. By [3] and [10], a derived equivalence between self-injective algebras induces a stable equivalence of Morita type.

In contrary to the case of derived equivalences, much less is known for stable equivalences of Morita type. Only recently, several authors began to study invariants for stable equivalences of Morita type. For example, it is proved that stable equivalences of Morita type preserve global dimension, stable center [1], stable Hochschild homology [6,7], stable Külshammer ideals [7,4] and stable cyclic homology [4], etc. In this note, we investigate the behavior of Hochschild cohomology under a stable equivalence of Morita type.

Hochschild cohomology of a finite dimensional algebra has a very rich structure. It has a cup product making it into a graded commutative algebra; it also has a Lie bracket of degree -1, making it into a graded Lie algebra. All these structures are known to be derived invariant (see [2,10]). For stable

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equivalences of Morita type, it has been proved previously that the dimensions of Hochschild cohomology groups of degree ≥ 1 [11] are preserved. It is also known that for self-injective algebras, the product structure of Hochschild cohomology is invariant as well [8].

Let k be a field. Recall first the definition of a stable equivalence of Morita type.

Definition 1.1. (Broué [1]) Let A and B be two finite dimensional k-algebras. Given two bimodules ${}_AM_B$ and ${}_BN_A$, we say that M and N induce a stable equivalence of Morita type between A and B, if ${}_AM_B$ and ${}_BN_A$ are projective as left modules and as right modules and there are bimodule isomorphisms:

$$ {}_AM \otimes_B N_A \simeq {}_AA_A \oplus {}_AP_A, \quad {}_BN \otimes_A M_B \simeq {}_BB_B \oplus {}_BQ_B $$

where ${}_AP_A$ and ${}_BQ_B$ are projective bimodules.

Let A be a finite dimensional k-algebra. Denote by $Z(A)$ the center of A and by $A^{\mathrm{e}} = A \otimes_k A^{\mathrm{op}}$ the enveloping algebra of A. All (left) modules are supposed to be finitely generated. The bounded derived category of left A-modules is denoted by $D^{\mathrm{b}}(A)$. It is well-known that $Z(A)$ can be identified with the space of homomorphisms of A-A-bimodules from ${}_AA_A$ to itself. Following Broué, we define the projective center $Z^{\mathrm{pr}}(A)$ to be the subspace of $Z(A)$ consisting of those homomorphisms from ${}_AA_A$ to itself which factor through projective A-A-bimodules. It is easy to see that the projective center is an ideal of $Z(A)$. The stable center $Z^{\mathrm{st}}(A)$ is the quotient of $Z(A)$ by $Z^{\mathrm{pr}}(A)$.

The Hochschild cohomology is defined as $HH^n(A) := \mathrm{Ext}^n_{A^{\mathrm{e}}}(A, A)$ for $n \geq 0$ and write $HH^*(A) = \oplus_{n\geq 0} HH^n(A)$. The cup-product gives a graded commutative algebra structure to the Hochschild cohomology. An easy fact is that $Z^{\mathrm{pr}}(A)$ is an ideal of $HH^*(A)$ as $Z^{\mathrm{pr}}(A)$ annihilates $\oplus_{n\geq 1} HH^n$. The quotient of $HH^*(A)$ by $Z^{\mathrm{pr}}(A)$ is called the stable Hochschild cohomology ring and is denoted by $HH^*_{\mathrm{st}}(A)$.

The main result of this note is the following.

Theorem 1.2. *Let A and B be two finite dimensional algebras which are stably equivalent of Morita type. Then their stable Hochschild cohomology rings are isomorphic as graded algebras.*

Some easy consequences of the main theorem are the results of Pogorzały and Xi.

Corollary 1.3. ([8, Theorem 1.1]) *Let A and B be two finite dimensional self-injective algebras which are stably equivalent of Morita type. Then their stable Hochschild cohomology are isomorphic as graded algebras.*

Corollary 1.4. ([11, Theorem 4.2]) *Let A and B be two finite dimensional algebras. If A and B are stably equivalent of adjoint type, then* $\dim HH^n(A) = \dim HH^n(B)$ *for all $n \geq 1$.*

The idea of the proof is to realize the stable Hochschild cohomology $HH^*_{\mathrm{st}}(A)$ as a quotient of the orbit algebra of A under the translation functor in the derived category of $A^{\mathrm{e}} = A \otimes_k A^{\mathrm{op}}$-modules.

2. A simple lemma and its applications. In this section, we shall present a simple lemma and some applications. The lemma is the key idea of the proof of our main result and it is in fact implicit in the work of Pogorzały [8,9].

Let $\mathcal{A}$ be a k-linear category with a k-linear endo-functor $\Sigma : \mathcal{A} \to \mathcal{A}$. Let X be an object of $\mathcal{T}$. Then we define a k-algebra structure on the vector space

$$\mathrm{End}^*_{(\mathcal{A},\Sigma)}(X) := \oplus_{n\geq 0}\mathrm{Hom}_{\mathcal{A}}(\Sigma^n X, X)$$

by putting $f \cdot g := f \circ \Sigma^n g$ where $f : \Sigma^n X \to X$ and $g : \Sigma^m X \to X$ are two morphisms in $\mathcal{A}$. It is obvious that with this product, $\mathrm{End}^*_{(\mathcal{A},\Sigma)}(X)$ becomes a graded ring with unit the identity morphism Id_X. If no confusion arises, we shall write $\mathrm{End}^*_{\mathcal{A}}(X)$ instead of $\mathrm{End}^*_{(\mathcal{A},\Sigma)}(X)$.

Let $\mathcal{A}'$ be another k-linear category with a k-linear endo-functor Σ'. Let $F : \mathcal{A} \to \mathcal{A}'$ be an additive functor such that there is a morphism of functors $\varphi : \Sigma' F \to F\Sigma$. Thus we have, for $n \geq 0$, morphisms of functors $\varphi^{(n)} : \Sigma'^n F \to F\Sigma^n$ defined by induction: $\varphi^{(1)} = \varphi$ and for $n \geq 2$, $\varphi^{(n)}$ is the composition

$$\Sigma'^n F = \Sigma'\Sigma'^{n-1} F \xrightarrow{\Sigma'\varphi^{(n-1)}} \Sigma' F\Sigma^{n-1} \xrightarrow{\varphi\Sigma^{n-1}} F\Sigma\Sigma^{n-1} = F\Sigma^n.$$

It is easy to see that for $n, m \geq 0$, we have $\varphi^{(n+m)} = \varphi^{(n)}\Sigma^m \circ \Sigma'^n\varphi^{(m)}$.

Then for an object $X \in \mathcal{A}$, F induces a map $\mathrm{End}^*_{\mathcal{A}}(X) \to \mathrm{End}^*_{\mathcal{A}'}(FX)$, denoted still by F, sending $f : \Sigma^n \to X$ to $Ff \circ \varphi^{(n)}_X$. A first result is the following

Lemma 2.1. *The map* $\mathrm{End}^*_{\mathcal{A}}(X) \to \mathrm{End}^*_{\mathcal{A}'}(FX)$ *is a homomorphism of graded algebras. If furthermore* F *is an equivalence and* φ *is an isomorphism of functors, then the above ring homomorphism is an isomorphism.*

Proof. We shall prove the first assertion, the second is clear. Now let X be an object of $\mathcal{A}$ and take two morphisms $f : \Sigma^n X \to X$ and $g : \Sigma^m X \to X$. We should prove that $Ff \circ \varphi^{(n)}_X \circ \Sigma'^n Fg \circ \Sigma'^n \varphi^{(m)}_X = Ff \circ F\Sigma^n g \circ \varphi^{(n+m)}_X$. This follows from the identity

$$\varphi^{(n+m)} = \varphi^{(n)}\Sigma^m \circ \Sigma'^n\varphi^{(m)}$$

and the commutative diagram

$$\begin{array}{ccc} \Sigma'^n F(\Sigma^m X) & \xrightarrow{\Sigma'^n Fg} & \Sigma'^n FX \\ \downarrow{\scriptstyle \varphi^{(n)}_{\Sigma^m X}} & & \downarrow{\scriptstyle \varphi^{(n)}_X} \\ F\Sigma^n\Sigma^m X & \xrightarrow[F\Sigma^n g]{} & F\Sigma^n(X) \end{array}$$

□

Remark 2.2. An application of Lemma 2.1 is [9, Theorem 1.1]. The statement is the following: Let A, B be self-injective, finite-dimensional algebras which are stably equivalent of Morita type, then $\mathrm{End}^*_{(A^e-\underline{\mathrm{mod}},\ \tau_{A^e})}(A)$ and $\mathrm{End}^*_{(B^e-\underline{\mathrm{mod}},\ \tau_{B^e})}(B)$ are isomorphic as algebras, where τ_{A^e} is the Auslander-Reiten translation.

The enveloping algebra A^e is a self-injective algebra. Denote by $\mathcal{A}$ (resp. $\mathcal{A}'$) the full subcategory of $A^e - \underline{\mathrm{mod}}$ (resp. $B^e - \underline{\mathrm{mod}}$) which consists of A-A

bimodules which are formed by the finite direct sum of $\tau^n_{A^e}(A)$ (resp. $\tau^n_{B^e}(B)$). The result then follows from two facts. The first fact is that

$$F = M \otimes_B - \otimes_B N : \mathcal{A}' \to \mathcal{A}$$

is an equivalence of categories and the second fact is that

$$M \otimes_B \tau^n_{B^e}(B) \otimes_B N \simeq \tau^n_{A^e}(A)$$

in $\mathcal{A}$.

Lemma 2.1 has a variant when the endo-functor Σ is an equivalence. We concentrate on the case when the categories in question are triangulated categories.

Let $\mathcal{T}$ be a k-triangulated category with translation functor [1] and let X be an object of $\mathcal{T}$. Then we define a ring structure over

$$\mathrm{End}^*_{\mathcal{T}}(X) := \oplus_{n\in\mathbb{Z}}\mathrm{Hom}_{\mathcal{T}}(X, X[n])$$

by imposing $fg := f[m] \circ g$ where $f : X \to X[n]$ and $g : X \to X[m]$ are two morphisms in $\mathcal{T}$. It is obvious that with this product, $\mathrm{End}^*_{\mathcal{T}}(X)$ becomes a graded ring. Remark that if $\mathcal{T} = D^b(A)$, then $\mathrm{End}^*_{\mathcal{T}}(A) = HH^*(A)$. Another remark is that if A is a self-injective algebra, then so is A^e and the stable category $\mathcal{T} = A^e$-$\underline{\mathrm{mod}}$ is triangulated. In this case, we have $\mathrm{End}^*_{\mathcal{T}}(A) = HH^*_{\mathrm{st}}(A)$.

Lemma 2.3. *Let $F : \mathcal{T} \to \mathcal{T}'$ be a k-triangle functor between k-triangulated categories such that $F[1] \simeq [1]F$. Then for an object $X \in \mathcal{T}$, F induced a homomorphism of rings $\mathrm{End}^*_{\mathcal{T}}(X) \to \mathrm{End}^*_{\mathcal{T}'}(FX)$, denoted still by F. If furthermore F is an equivalence, then the above ring homomorphism is an isomorphism.*

3. Proof of the theorem. In this section, we prove the main theorem of this note using Lemma 2.3.

Suppose that there is a stable equivalence of Morita type $({}_AM_B, {}_BN_A)$ between two finite dimensional algebras A and B. Since if A is separable, then it is projective over A^e and thus $HH^*_{\mathrm{st}}(A) = 0$, we can assume that A and B have no separable summands and by [5, Theorem 2.2], one can even assume that A and B are indecomposable. Since M_B and ${}_BN$ are projective, we have the triangle functor

$$F = M \otimes_B - \otimes_B N : D^{\mathrm{b}}(B^e) \to D^{\mathrm{b}}(A^e)$$

and similarly the triangle functor

$$G = N \otimes_A - \otimes_A M : D^{\mathrm{b}}(A^e) \to D^{\mathrm{b}}(B^e).$$

Obviously they commute with the translation functors.

We shall define two homomorphisms of algebras

$$t_F : HH^*_{\mathrm{st}}(B) \to HH^*_{\mathrm{st}}(A)$$

and

$$t_G : HH^*_{\mathrm{st}}(A) \to HH^*_{\mathrm{st}}(B)$$

and prove that $t_F \circ t_G$ and $t_G \circ t_F$ are isomorphisms, which will complete the proof.

Since $F(B) = M \otimes_B B \otimes_B N \simeq A \oplus P$, we choose an injection $i : A \to F(B)$ and a projection $p : F(B) \to A$ such that $p \circ i = Id_A$. One can then construct a map

$$\mathrm{End}^*_{D^{\mathrm{b}}(A^{\mathrm{e}})}(F(B)) \to \mathrm{End}^*_{D^{\mathrm{b}}(A^{\mathrm{e}})}(A) = HH^*(A)$$

which sends $f \in \mathrm{Hom}_{D^{\mathrm{b}}(A^{\mathrm{e}})}(F(B), F(B)[n])$ to $p[n] \circ f \circ i$. This map is not a homomorphism of algebras in general, but we prove the following.

Claim 1. *The composition of*

$$\mathrm{End}^*_{D^{\mathrm{b}}(A^{\mathrm{e}})}(F(B)) \to \mathrm{End}^*_{D^{\mathrm{b}}(A^{\mathrm{e}})}(A) = HH^*(A)$$

with the natural surjection $HH^*(A) \to HH^*_{\mathrm{st}}(A)$ is a homomorphism of algebras. The composition will be denoted by $\mathrm{End}_{D^{\mathrm{b}}(A^{\mathrm{e}})}(F(B)) \xrightarrow{C} HH^*_{\mathrm{st}}(A)$.

Proof. Given a morphism $f \in \mathrm{Hom}_{D^{\mathrm{b}}(A^{\mathrm{e}})}(F(B), F(B)[n])$ for some $n \in \mathbb{N}$ and identify $F(B)$ with $A \oplus P$, f is presented by a matrix $f = \begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix}$. The composition $\mathrm{End}_{D^{\mathrm{b}}(A^{\mathrm{e}})}(F(B)) \xrightarrow{C} HH^*_{\mathrm{st}}(A)$ sends f to $[f_1]$, where $[f_1]$ is the residue class of f_1 in $HH^*_{\mathrm{st}}(A)$.

Now let $g = \begin{pmatrix} g_1 & g_2 \\ g_3 & g_4 \end{pmatrix} \in \mathrm{Hom}_{D^{\mathrm{b}}(A^{\mathrm{e}})}(F(B), F(B)[m])$. Then the image of

$$g \cdot f = g[n] \circ f = \begin{pmatrix} g_1[n] \circ f_1 + g_2[n] \circ f_3 & * \\ * & * \end{pmatrix}$$

is $[g_1[n] \circ f_1 + g_2[n] \circ f_3] = [g_1[n] \circ f_1] = [g_1][f_1]$. This proves that the composition is a homomorphism of algebras. □

Now look at the following diagram

$$\begin{array}{ccc} \mathrm{End}^*_{D^{\mathrm{b}}(B^{\mathrm{e}})}(B) = HH^*(B) & \xrightarrow{F} & \mathrm{End}^*_{D^{\mathrm{b}}(A^{\mathrm{e}})}(F(B)) \\ \downarrow & & \downarrow C \\ HH^*_{\mathrm{st}}(B) & \overset{t_F}{\dashrightarrow} & HH^*_{\mathrm{st}}(A) \end{array}$$

In this diagram, the upper morphism comes from the preceding lemma; the leftmost morphism is the natural surjection; the rightmost morphism is the homomorphism of algebras constructed above. Now we prove

Claim 2. *The bottom morphism in the preceding diagram exists and is a homomorphism of algebras, which is denoted by t_F; the morphism t_G can be defined similarly.*

Proof. It suffices to prove that the projective center $Z^{\mathrm{pr}}(B)$ lies in the kernel of the composition

$$\mathrm{End}^*_{D^{\mathrm{b}}(B^{\mathrm{e}})}(B) \xrightarrow{F} \mathrm{End}^*_{D^{\mathrm{b}}(A^{\mathrm{e}})}(F(B)) \xrightarrow{C} HH^*_{\mathrm{st}}(A).$$

Let $f : B \to B$ be a homomorphism of bimodules which factor through a projective bimodule R. Then the image of f is the residue class of the composition

$$A \xrightarrow{i} F(B) \to F(R) \to F(B) \xrightarrow{p} A$$

which is zero since $F(R) = M \otimes_B R \otimes_B N$ is a projective A-A-bimodule. □

Now we should prove that the compositions $t_F \circ t_G$ and $t_G \circ t_F$ are isomorphisms.

We know that $GF(B)$ have two decompositions:

$$GF(B) = (N \otimes_A M) \otimes_B B \otimes_B (N \otimes_A M) \simeq (B \oplus Q) \otimes_B (B \oplus Q) \simeq B \oplus Q_1$$

for some projective bimodule Q_1 and also

$$GF(B) \simeq N \otimes_A (A \oplus P) \otimes_A M \simeq (N \otimes_A A \otimes_A M) \oplus \cdots \simeq B \oplus Q_2$$

for some projective bimodule Q_2. In the first decomposition, note $i_I : B \to GF(B)$ to be the injection and $p_I : GF(B) \to B$ the projection, similarly for i_{II} and p_{II}. The key observation is the following

Claim 3. *There are isomorphisms of bimodules $\theta : B \to B$ and $\tau : B \to B$ such that given a morphism $f : GF(B) \to GF(B)[n]$ in $D^{\mathrm{b}}(B^{\mathrm{e}})$, then we have*

$$p_{II}[n] \circ f \circ i_{II} = \theta[n] \circ p_I[n] \circ f \circ i_I \circ \tau \in HH^*_{\mathrm{st}}(B).$$

Proof. Consider the isomorphisms given by the two decompositions of $GF(B)$:

$$B \oplus Q_1 \xrightarrow{\simeq} GF(B) \xrightarrow{\simeq} B \oplus Q_2.$$

This map is given by

$$g = \begin{pmatrix} g_1 & g_2 \\ g_3 & g_4 \end{pmatrix}$$

and denote its inverse

$$h = \begin{pmatrix} h_1 & h_2 \\ h_3 & h_4 \end{pmatrix}.$$

Since $g \circ h = \mathrm{Id}$, we have $g_1 \circ h_1 + g_2 \circ h_3 = \mathrm{Id}_B \in \mathrm{End}_{B^{\mathrm{e}}}(B)$. Since B is indecomposable and non separable, and $g_2 \circ h_3$ factors through a projective bimodule, $g_2 \circ h_3 \in \mathrm{RadEnd}_{B^{\mathrm{e}}}(B)$ and thus $g_1 \circ h_1 = \mathrm{Id}_B - g_2 \circ h_3$ is invertible. Dually, $h_1 \circ g_1$ is invertible. We deduce that $\theta := g_1$ and $\tau := h_1$ are invertible.

Now take $f : GF(B) \to GF(B)[n]$ and identify $GF(B)$ with $B \oplus Q_1$ using the first decomposition. Then f can be presented by a matrix $f = \begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} : B \oplus Q_1 \to (B \oplus Q_1)[n]$. If we express f using the second decomposition, then it is presented by the matrix

$$\begin{pmatrix} g_1[n] & g_2[n] \\ g_3[n] & g_4[n] \end{pmatrix} \begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} \begin{pmatrix} h_1 & h_2 \\ h_3 & h_4 \end{pmatrix}$$

and therefore

$$\begin{aligned} &p_{II}[n] \circ f \circ i_{II} \\ &= p_{II}[n] \circ \begin{pmatrix} g_1[n] & g_2[n] \\ g_3[n] & g_4[n] \end{pmatrix} \begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} \begin{pmatrix} h_1 & h_2 \\ h_3 & h_4 \end{pmatrix} \circ i_{II} \\ &= g_1[n] \circ f_1 \circ h_1 + g_2[n] \circ f_3 \circ h_1 + g_1[n] \circ f_2 \circ h_3 + g_2[n] \circ f_4 \circ h_3 \\ &= \theta[n] \circ p_I[n] \circ f \circ i_I \circ \tau \in HH^*_{\mathrm{st}}(B). \end{aligned}$$

□

Therefore, the two homomorphisms of algebras

$$\operatorname{End}^*_{D^{\mathrm{b}}(B^{\mathrm{e}})}(GF(B)) \xrightarrow{I} HH^*_{\mathrm{st}}(B)$$

constructed in Claim 1 using the first decomposition and

$$\operatorname{End}^*_{D^{\mathrm{b}}(B^{\mathrm{e}})}(GF(B)) \xrightarrow{II} HH^*_{\mathrm{st}}(B)$$

constructed in Claim 1 using the second decomposition differ by an isomorphism of graded vector spaces

$$\varphi : HH^*_{\mathrm{st}}(B) \xrightarrow{\simeq} HH^*_{\mathrm{st}}(B)$$

which sends $u : B \to B[n]$ to $\theta[n] \circ u \circ \tau$, as θ and τ are invertible.

Now we consider the following diagram

$$\begin{array}{ccccccc}
\operatorname{End}^*_{D^{\mathrm{b}}(B^{\mathrm{e}})}(B) & \xrightarrow{F} & \operatorname{End}^*_{D^{\mathrm{b}}(A^{\mathrm{e}})}(F(B)) & \xrightarrow{G} & \operatorname{End}^*_{D^{\mathrm{b}}(B^{\mathrm{e}})}(GF(B)) & \longrightarrow & \operatorname{End}^*_{D^{\mathrm{b}}(B^{\mathrm{e}})}(B) \\
\downarrow & & \downarrow & & \downarrow & \searrow^{I} & \downarrow \\
 & & \operatorname{End}^*_{D^{\mathrm{b}}(A^{\mathrm{e}})}(A) & \xrightarrow{G} & \operatorname{End}^*_{D^{\mathrm{b}}(B^{\mathrm{e}})}(G(A)) & & \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
HH^*_{\mathrm{st}}(B) & \xrightarrow{t_F} & HH^*_{\mathrm{st}}(A) & \xrightarrow{t_G} & HH^*_{\mathrm{st}}(B) & \xrightarrow[\simeq]{\psi} & HH^*_{\mathrm{st}}(B)
\end{array}$$

The composition of homomorphisms in the second column is the morphism

$$\operatorname{End}^*_{D^{\mathrm{b}}(A^{\mathrm{e}})}(F(B)) \xrightarrow{C} HH^*_{\mathrm{st}}(A)$$

and the composition of homomorphisms in the third column is the morphism

$$\operatorname{End}^*_{D^{\mathrm{b}}(B^{\mathrm{e}})}(GF(B)) \xrightarrow{II} HH^*_{\mathrm{st}}(B),$$

and ψ is the inverse of φ.

The commutativity of this diagram is clear. We now prove

Claim 4. *The composition of all homomorphisms in the upper row is the identity.*

Proof. This follows from the isomorphism of functors

$$GF \simeq N \otimes_A M \otimes_B - \otimes_B N \otimes_A M \simeq (B \oplus Q) \otimes_B - \otimes_B (B \otimes Q) = \mathrm{Id} \oplus \cdots .$$

□

Now $\varphi \circ t_G \circ t_F = Id$, thus $t_G \circ t_F$ is invertible. Dually we have $t_F \circ t_G$ is invertible. We deduce that t_F and t_G are isomorphisms of algebras. This completes the proof of the main theorem. Notice that this also proves that φ is an isomorphism of algebras, and thus $\tau = \theta^{-1}$.

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