

实验一 补充

常见空间曲线和曲面
(标准方程及Matlab作图)

主要内容

- 球面、椭球面
- 单叶双曲面与双叶双曲面
- 椭圆抛物面与双曲抛物面
- 圆柱螺线、圆锥螺线与抛物螺线
- Matlab 绘图

球面

- 球面标准方程 (以原点为球心)

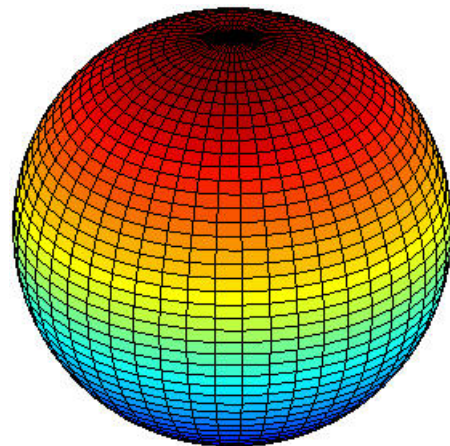
$$x^2 + y^2 + z^2 = R^2 \quad (R > 0)$$

$$\begin{cases} x = R \cdot \sin \varphi \cdot \cos \theta \\ y = R \cdot \sin \varphi \cdot \sin \theta \\ z = R \cdot \cos \varphi \end{cases}$$

$$\begin{aligned} 0 \leq \theta < 2\pi \\ 0 \leq \varphi \leq \pi \end{aligned}$$

经度

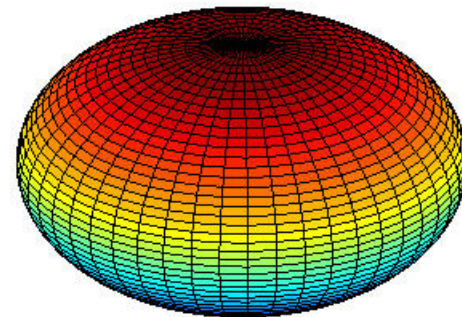
纬度



椭球面

● 椭球面标准方程

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad (a, b, c > 0)$$



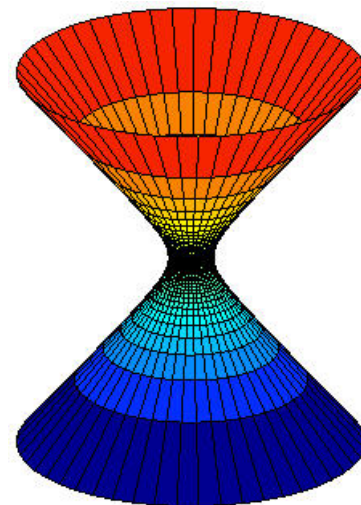
$$\begin{cases} x = a \cdot \sin \varphi \cdot \cos \theta \\ y = b \cdot \sin \varphi \cdot \sin \theta \\ z = c \cdot \cos \varphi \end{cases}$$

$$\begin{aligned} 0 &\leq \theta < 2\pi \\ 0 &\leq \varphi \leq \pi \end{aligned}$$

单叶双曲面

● 单叶双曲面标准方程

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \quad (a, b, c > 0)$$



$$\begin{cases} x = a \cdot \sec \varphi \cdot \cos \theta \\ y = b \cdot \sec \varphi \cdot \sin \theta \\ z = c \cdot \tan \varphi \end{cases}$$

$$\begin{aligned} 0 &\leq \theta < 2\pi \\ -\pi/2 &< \varphi < \pi/2 \end{aligned}$$

双叶双曲面

● 双叶双曲面标准方程

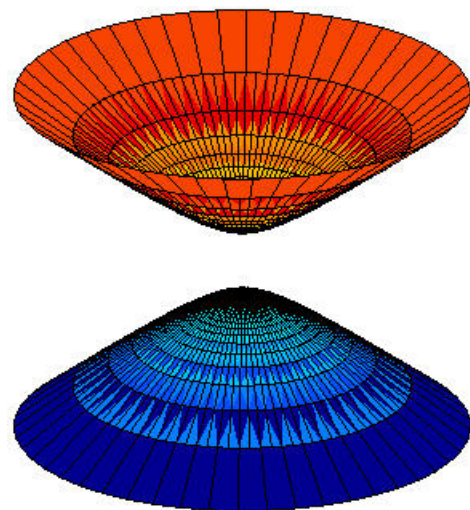
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

$$(a, b, c > 0)$$

$$\begin{cases} x = a \cdot \tan \varphi \cdot \cos \theta \\ y = b \cdot \tan \varphi \cdot \sin \theta \\ z = c \cdot \sec \varphi \end{cases}$$

$$0 \leq \theta < \pi$$

$$0 < \varphi < 2\pi, \quad \varphi \neq \frac{1}{2}\pi, \frac{3}{2}\pi$$

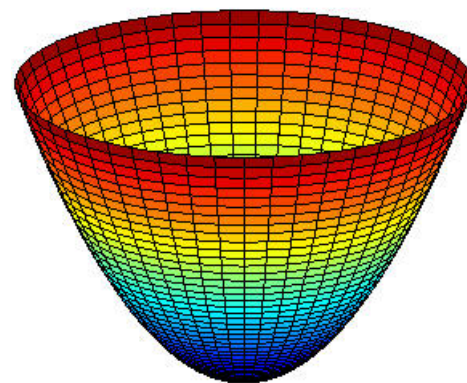


椭圆抛物面

- 椭圆抛物面标准方程

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2z \quad (a, b > 0)$$

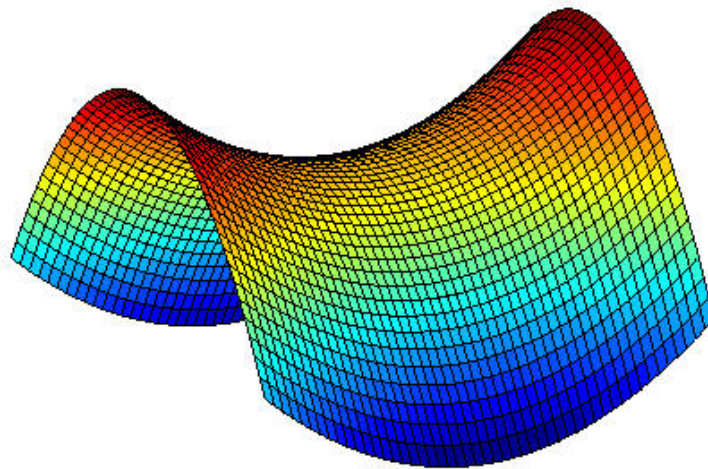
$$\begin{cases} x = r \cdot a \cdot \cos \theta \\ y = r \cdot b \cdot \sin \theta \\ z = r^2/2 \end{cases} \quad \begin{cases} 0 \leq \theta < 2\pi \\ r > 0 \end{cases}$$



双曲抛物面

- 双曲抛物面标准方程

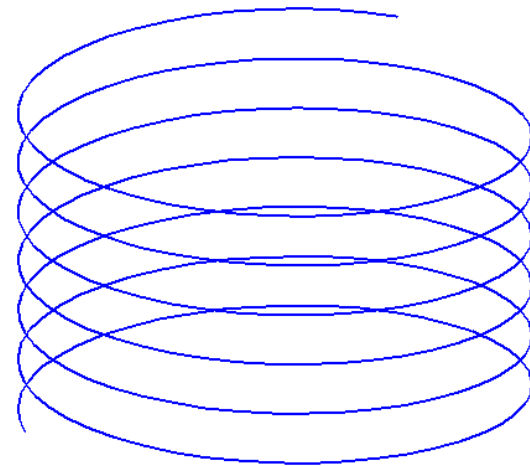
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z \quad (a, b > 0)$$



圆柱螺线圆锥螺线

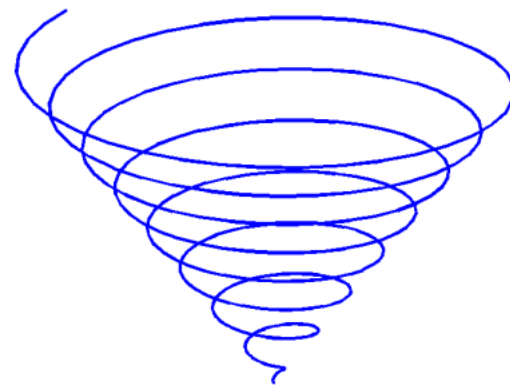
● 圆柱螺线标准方程

$$\begin{cases} x = a \cdot \cos t \\ y = a \cdot \sin t \\ z = b \cdot t \end{cases} \quad (-\infty < t < +\infty)$$



● 圆锥螺线标准方程

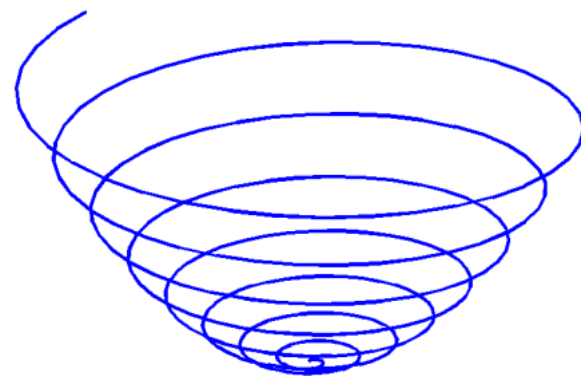
$$\begin{cases} x = a \cdot t \cdot \cos t \\ y = b \cdot t \cdot \sin t \\ z = c \cdot t \end{cases} \quad (0 < t < +\infty)$$



抛物螺线

- 轴截面的曲边为一条抛物线的螺线

$$\begin{cases} x = a \cdot t \cdot \cos t \\ y = b \cdot t \cdot \sin t \\ z = c \cdot t^2 \end{cases} \quad 0 < t < +\infty$$



易知该螺线位于下面的抛物面上

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$

球面的绘制

- 法一、利用球面的参数方程数值作图: **surf**

$$\begin{cases} x = R \cdot \sin \varphi \cdot \cos \theta \\ y = R \cdot \sin \varphi \cdot \sin \theta \\ z = R \cdot \cos \varphi \end{cases} \quad \begin{array}{l} 0 \leq \theta < 2\pi \\ 0 \leq \varphi \leq \pi \end{array}$$

```
u=[0:pi/60:2*pi]; v=[0:pi/60:pi];  
[U,V]=meshgrid(u,v);  
R=3;  
X=R*sin(V).*cos(U);  
Y=R*sin(V).*sin(U);  
Z=R*cos(V);  
surf(X,Y,Z);  
axis equal;
```

球面的绘制

- 法二、利用球面的参数方程符号作图：ezsurf

$$\begin{cases} x = R \cdot \sin \varphi \cdot \cos \theta \\ y = R \cdot \sin \varphi \cdot \sin \theta \\ z = R \cdot \cos \varphi \end{cases} \quad \begin{cases} 0 \leq \theta < 2\pi \\ 0 \leq \varphi \leq \pi \end{cases}$$

```
ezsurf('3*sin(u)*cos(v)', ...  
       '3*sin(u)*sin(v)', '3*cos(u)', ...  
       [0,pi, 0,2*pi]);
```

第一自变量的取值范围

第二自变量的取值范围

按字母顺序

符号作图无需
用数组运算

球面的绘制

- 法三、利用 **sphere** 函数数值作图

```
[X,Y,Z]=sphere(60);  
R=3;  
X=R*X;  
Y=R*Y;  
Z=R*Z;  
surf(X,Y,Z);
```

注：该方法不推荐！

椭球面的绘制

$$\begin{cases} x = a \cdot \sin \varphi \cdot \cos \theta \\ y = b \cdot \sin \varphi \cdot \sin \theta \\ z = c \cdot \cos \varphi \end{cases} \quad \begin{array}{l} 0 \leq \theta < 2\pi \\ 0 \leq \varphi \leq \pi \end{array}$$

例：绘制 $a=3$, $b=3$, $c=2$ 时的图像

```
ezsurf('3*sin(u)*cos(v)', ...  
       '3*sin(u)*sin(v)', '2*cos(u)', ...  
       [0,pi, 0,2*pi]);
```

单叶双曲面的绘制

$$\begin{cases} x = a \cdot \sec \varphi \cdot \cos \theta \\ y = b \cdot \sec \varphi \cdot \sin \theta \\ z = c \cdot \tan \varphi \end{cases} \quad \begin{array}{l} 0 \leq \theta < 2\pi \\ -\frac{\pi}{2} < \varphi < \frac{\pi}{2} \end{array}$$

例：绘制 $a=3$, $b=3$, $c=5$ 时的图像

```
ezsurf('3*sec(u)*cos(v)', ...  
       '3*sec(u)*sin(v)', '5*tan(u)', ...  
       [-pi/2, pi/2, 0, 2*pi]);
```

双叶双曲面的绘制

$$\begin{cases} x = a \cdot \tan \varphi \cdot \cos \theta \\ y = b \cdot \tan \varphi \cdot \sin \theta \\ z = c \cdot \sec \varphi \end{cases}$$

$$0 \leq \theta < \pi$$

$$0 < \varphi < 2\pi, \quad \varphi \neq \frac{1}{2}\pi, \frac{3}{2}\pi$$

例：绘制 $a=3$, $b=4$, $c=5$ 时的图像

```
ezsurf('3*tan(u)*cos(v)', ...  
       '3*tan(u)*sin(v)', '5*sec(u)', ...  
       [0,2*pi, 0,pi]);
```

椭圆抛物面的绘制

$$\begin{cases} x = a \cdot r \cdot \cos \theta \\ y = b \cdot r \cdot \sin \theta \\ z = 0.5 r^2 \end{cases} \quad \begin{array}{l} 0 \leq \theta < 2\pi \\ r > 0 \end{array}$$

例： 绘制 $a=2$, $b=3$ 时的图像

```
ezsurf('2*r*cos(u)', '3*r*sin(u)', ...  
       'r*r/2', [0,10, 0,2*pi]);
```

双曲抛物面的绘制

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z \quad (a, b > 0)$$

例：绘制 $a=4$, $b=3$ 时的图像

```
ezsurf('(x^2/4^2-y^2/3^2)/2', [-4,4, -3,3]);
```

圆柱螺线的绘制

$$\begin{cases} x = a \cdot \cos t \\ y = a \cdot \sin t \\ z = b \cdot t \end{cases} \quad -\infty < t < +\infty$$

例：绘制 $a=3$, $b=5$, $0 < t < 50$ 时的图像

```
ezplot3('3*cos(t)', '3*sin(t)', '5*t', [0, 50]);
```

圆锥螺线的绘制

$$\begin{cases} x = a \cdot t \cdot \cos t \\ y = b \cdot t \cdot \sin t \\ z = c \cdot t \end{cases} \quad 0 < t < +\infty$$

例：绘制 $a=3$, $b=2$, $c=3$, $0 < t < 50$ 时的图像

```
ezplot3('2*t*cos(t)', '2*t*sin(t)', '3*t', ...  
        [0,50]);
```

抛物螺线的绘制

- 轴截面的曲边为抛物线的螺线

$$\begin{cases} x = a \cdot t \cdot \cos t \\ y = b \cdot t \cdot \sin t \\ z = c \cdot t^2 \end{cases} \quad 0 < t < +\infty$$

例： 绘制 $a=2$, $b=2$, $c=1/3$, $0 < t < 50$ 时的图像

```
ezplot3('2*t*cos(t)', '2*t*sin(t)', 't^2/3', ...  
        [0, 50]);
```