Summer School Schedule

	7.15	7.16	7.17	7.18	7.19	7.20	7.21	7.22	7.23
8:45–9:45		Webster	Wang	Juteau	Break	Wang	Juteau	Webster	Juteau
10:00–11:00		Webster	Wang	Juteau	Break	Wang	Juteau	Webster	Juteau
13:30–14:30	Wang	Juteau	Webster	Wang	Break	Webster	Wang	Juteau	Webster
14:45–15:45	Wang								
16:00–17:00	Webster								

Venue: Room 110, Teaching Building 4, Minghang Campus, ECNU.

Workshop Schedule

	7/25	7/26	7/27
09:00-10:00	Shun-Jen Cheng	Fan Qin	Chongying Dong
10:00-10:20	Break	Photo	Break
10:20-11:20	Husileng Xiao	Weiqiang Wang	Cuibo Jiang
	Lunch	Lunch	Lunch
13:30-14:30	Daniel Juteau	Toshiyuki Tanisaki	Peng Shan
14:40-15:40	Ben Webster	Ming Lu	Naihuan Jing
15:40-16:00	Break	Break	Break
16:00-17:00	Huanchen Bao	Hideya Watanabe	Linliang Song

Venue: Room 110, Teaching Building 4, Minghang Campus, ECNU.

Summer School

Course 1: Representations of rational Cherednik algebras

Daniel Juteau

Rational Cherednik algebras were introduced by Etingof and Ginzburg in their landmark 2002 paper on symplectic reflection algebras. They are (double) degenerations of the double affine Hecke algebras (DAHAs) introduced by Cherednik in the 1990's to solve the Macdonald conjectures. While DAHAs make sense only for Weyl groups, the rational Cherednik algebras do make sense for finite Coxeter groups, and even for complex reflection group: those are generalizations of finite Coxeter groups to the complex case, with reflections allowed to have any order, rather than just order 2. Given a complex reflection group W acting on a space V, the associated rational Cherednik algebra $H_c(W, V)$ can be defined by a faithful representation on the polynomial algebra $\mathbb{C}[V] = S(V^*)$: it is generated by polynomials acting by multiplication, the group algebra of W, and the Dunkl operators, which are a deformation of the differential operators depending on some parameters c_r (one for each conjugacy classes of reflections in W). We will review those definitions, and the key feature of Dunkl operators: they commute. We have a Poincaré-Birkhoff-Witt theorem, and hence a triangular decomposition of the algebra, analogous to that of the enveloping algebra of a semisimple Lie algebra.

Pursuing this analogy, we have a category O, which is a highest weight category, with standard modules parametrized by the irreducible representations of W. Moreover, we have an exact functor (the Khnizhnik-Zamolodchikov functor, introduced by Ginzburg-Guay-Opdam-Rouquier), from that category to the category of modules for the finite Hecke algebra; this is an analogue of Soergel's functor $\mathbb V$ in the theory of semisimple Lie algebras. I will also explain the Bezrukavnikov-Etingof induction and restriction functors and their properties. Finally, I will discuss some aspects of one of the main problems, namely the determination of the supports of simple modules, and in particular the determination of the parameters for which simple modules become finite dimensional. For example I will discuss Etingof's determination of the support of the spherical module (the unique simple quotient of the polynomial representation).

Course 2: Quantum symmetric pairs and applications

Weiqiang Wang

Since its introduction by Drinfeld and Jimbo in 1980s, the theory of quantum groups has found many developments and applications, including R-matrices, canonical bases, q-Schur duality, connections to Hall algebras, geometry of type A flag varieties and quiver varieties, modular representations, Kazhdan-Lusztig theory of (super) type A, and categorification. Quantum symmetric pairs (U,U^i) are quantizations of the symmetric pairs (g,g^i), where i is an involution on a simple Lie algebra g. The symmetric pairs are in bijection with the real simple Lie algebras. In recent years, it has become increasingly clear (thanks to Huanchen Bao and others) that the quantum symmetric pairs provide a right setting for natural generalizations of the aforementioned constructions and applications of quantum groups.

Here is a detailed description of the mini-course:

(1) We review the quantum group associated to a simple Lie algebra g, with an emphasis on the type A case. We explain the q-Schur duality between U and Hecke algebra for the symmetric group.

(1-i) We introduce an example of quantum symmetric pair of type AIII, (U,U^i), where U = U_q(sl_N). We shall explain an i-Schur duality between U^i and Hecke algebra of type B (of equal or unequal parameter).

(2) We review the canonical bases on half a quantum group and its finite dimensional simple modules. We introduce the quasi-R matrix of Lusztig, and use it to define canonical bases of tensor product U-modules.

(2-i) We introduce a bar involution on U^i and a generalization of quasi-R matrix for (U,U^i). We establish a new canonical basis (called ı-canonical basis) for tensor product U-modules when viewed as U^i-modules.

(3) We explain how to view the BGG category O of Lie algebra of type A (and its super analogue) as a categorification of the Schur duality, and how the Kazhdan-Lusztig theory for O can be reformulated in terms of canonical basis.

(3-i) We explain how the BGG category O^i of Lie algebras of type B/C/D (and its super analogue) as a categorification of the ı-Schur duality, and how the Kazhdan-Lusztig theory for O^i can be reformulated in terms of i-canonical basis.

(4) Time permitting, we review the construction of canonical basis on the modified quantum group \dot U, and explain the construction of i-canonical basis on the modified form of U^i. Alternatively, we may discuss some geometric constructions.

Course 3: Categorification in representation theory: Heisenberg and Kac-Moody

Ben Webster

This course will be an introduction to using categorifications of Lie algebras, especially sl_n and its affine generalization, in studying representation theory. Our starting point will be the notion of a categorical Heisenberg action of arbitrary level. This is, not so surprisingly, an action of a monoidal category which induces an action of the usual Heisenberg algebra on the level of Grothendieck groups.

These are easy to define and give examples of: representations of symmetric groups, general linear Lie (super)algebras and Cherednik algebras all give examples.

Heisenberg categorifications have a powerful internal structure, however: based on work of Vershik-Okounkov and Brundan-Kleshchev, one can show that they induce not just an action of the Heisenberg algebra, but of an affine Lie algebra. This will lead us to the definition of the categorified universal enveloping algebra of Khovanov, Lauda and Rouquier, which in turn supplies new insight into the representation theory of all the examples discussed above.

Workshop

The amplituhedron

Huanchen Bao

An amplituhedron (defined in terms of totally nonnegative Grassmannian) is a geometric structure introduced in 2013 by Nima Arkani-Hamed and Jaroslav Trnka. It enables simplified calculation of particle interactions in some quantum field theories. In this talk, I will explain the main motivational conjecture and some current progress. This is based on work in progress with Xuhua He.

Irreducible characters of exceptional Lie superalgebras in the BGG category

Shun-Jen Cheng

We explain a solution to the irreducible character problem for the finite-dimensional exceptional simple Lie superalgebras D(2,1,\zeta) and G(3) in the BGG category. This is a joint work with Weiqiang Wang.

Vertex operator superalgebras and 16 fold way conjecture

Chongying Dong

Let V=V_0+V_1 be a vertex operator superalgebra. Then V has a canonical automorphism σ of order 2 from the super structure. We assume V is rational and C_2-cofinite. We will discuss the super σ-twisted modules, connection between representations of V and V_0. We will also explain how the representation theory of V is related to the 16-fold way conjecture in category theory. This talk is based on joint work with Richard Ng and Li Ren.

Level-rank duality for orthogonal affine vertex operator algebras

Cuibo Jiang

We will talk about relations between coset vertex operator algebras and tensor decompositions of orthogonal affine vertex operator algebras, which present a version of level-rank duality. This talk is based on joint work with Ching-Hung Lam.

Presentation of Yangian algebras in BCD types

Naihuan Jing

It is well-known that the R-matrix presentation of the Yangian in type A yields generators of its Drinfeld presentation. It has been an open problem since Drinfeld's pioneering work to extend this result to the remaining types. We will provide a solution for the classical types of BCD by constructing an explicit isomorphism between the R-matrix and Drinfeld presentations of the Yangian. It is based on an embedding theorem which allows us to consider the Yangian of rank n-1 as a subalgebra of the Yangian of rank n of the same type. This is joint work with A. Molev and M. Liu.

Support of the spherical module of the rational Cherednik algebra

Daniel Juteau

I will explain a criterion obtained in joint work with Stephen Griffeth giving the support of the spherical module, which is the unique quotient of the polynomial representation of the rational Cherednik algebra associated to an arbitrary complex reflection group and arbitrary parameter c. In particular, this determines for which parameters the spherical module is finite dimensional. This extends and simplifies previous work of Varagnolo-Vasserot and Etingof. To get the most concrete form of the result, however, we need to assume some hypotheses on the Hecke algebras of complex reflection groups that are not yet proved in full generality (but are known in particular for Coxeter groups and for G(d,1,n)).

Modified Ringel-Hall algebras and quantum symmetric pairs

Ming Lu

In 1990, C. M. Ringel introduced Hall algebras for quiver representations to realise the positive part of the corresponding quantum group. From that time, many people have tried to realise quantum groups by using Hall algebras. In 2013, T. Bridgeland solved this problem completely by constructing a kind of Hall algebras for the exact categories of Z/2-graded complexes of an a hereditary abelian category with enough projective objects. In 2016, Liangang Peng and me generalised Bridgeland's construction to arbitary hereditary abelian categories, and defined the so called modified Ringel-Hall algebras.

Recently, quantum symmetric pairs become hot in Lie theory, quantum groups. A natural question is how to construct a kind of Hall algebras to categorify the quantum symmetric pairs. In order to solve this problem, first, we construct modified Ringel-Hall algebras for 1-Gorenstein algebras, and prove that it is invariant by taking derived equivalences. Second, we construct a kind of 1-Gorenstein algebras from quivers with involutions. By considering modified Ringel-Hall algebras of this kind of 1-Gorenstein algebras, we give a realization of the quantum symmetric pairs of finite type. Third, using the APR-tilting transformation of this kind of 1-Gorenstein algebras, we realise the braid group actions of the quantum symmetric pairs obtained by S. Kolb and J. Pellegrini.

This is an ongoing work joint with Weiqiang Wang.

Bases of cluster algebras

Fan Qin

In this talk, we give a review of cluster algebras and their bases. We present and compare different bases of cluster algebras arising from representation theory, categorification, and geometry. In the end, we discuss some recent progress in this topic.

G_1T-modules and affine Springer fibres

Peng Shan

We will explain some relationships between the center of G_1T-modules and the cohomol ogy of certain affine Springer fibres. This is based on a joint project in progress with Bezr ukavnikov, Qi and Vasserot.

Affine Brauer category and parabolic category O in types BCD

Linliang Song

A strict monoidal category referred to as affine Brauer category AB is introduced over a commutative ring κ containing multiplicative identity 1 and invertible element 2. We prove that morphism spaces in AB are free over κ. The cyclotomic (or level k) Brauer category $CB^f(\omega)$ is a quotient category of AB. We prove that any morphism space in $CB^f(\omega)$ is free over κ with maximal rank if and only if the $\mathbf u$-admissible condition holds. Affine Nazarov-Wenzl algebras and cyclotomic Nazarov-Wenzl algebras will be realized as certain endomorphism algebras in AB and $CB^f(\omega)$, respectively. We will establish higher Schur-Weyl duality between cyclotomic Nazarov-Wenzl algebras and parabolic BGG categories $\mathcal O$ associated to symplectic and orthogonal Lie algebras over the complex field $\mathbb C$. This enables us to use standard arguments to compute decomposition matrices of cyclotomic Nazarov-Wenzl algebras. The level two case was considered by Ehrig and Stroppel. This is a joint work with Hebing Rui.

Quantized flag manifolds and representations of the quantum groups

Toshiyuki Tanisaki

The quantized flag manifold is a non-commutative scheme, which is an object in the non-commutative algebraic geometry. We consider localization of the representations of the quantum group on the quantized flag manifold, and give analogues of the results of Beilinson-Bernstein and Bezrukavnikov-Mirkovic-Rumynin.

Perspectives on quantum symmetric pairs

Weiqiang Wang

There are several major constructions for quantum groups, which have connections and applications to diverse areas of mathematics. A partial list includes Schur-Jimbo duality, BGG category O, canonical bases, R-matrices, quiver varieties, Hall algebras, categorification, and modular representation theory (many of which were originated by Lusztig, after earlier works of Drinfeld and Jimbo). In recent years, it has emerged that many of these constructions can be generalized to the setting of quantum symmetric pairs, and often the original quantum group constructions can be viewed in the context of quantum symmetric pairs of diagonal type (or trivial pairs). In this talk we will provide an overview on some of these recent developments on quantum symmetric pairs, and then indicate some new directions which are being developed.

Global crystal bases for integrable modules over a quantum symmetric pair of type AIII

Hideya Watanabe

Quantum symmetric pairs (QSPs) appear in many areas of mathematics and physics such as representation theory, low-dimensional topology, and integrable system. Especially, QSPs of type AIII are known to have a deep connection with the representation theory of Hecke algebras. In this talk, we introduce the notion of global crystal bases for integrable modules over a QSP of type AIII, and show some applications in the representation theory.

Cherednik algebras and Coulomb branches

Ben Webster

We'll explain how a different presentation of the Cherednik algebras of G(r,1,n), emphasizing the role of Dunkl-Opdam operators, allows us to give an explicit presentation of, and grading on, category O for these algebras, as well as a more algebraic description of induction, restriction and the KZ functor. This is one path that leads to a proof of Rouquier's conjecture on the decomposition numbers of category O's. This is actually a special case of a more general phenomenon where presenting algebras in the framework of Coulomb branches (as defined by Braverman, Finkelberg and Nakajima) leads to a fruitful analysis of their representation theory.

Finite W super algebra via super Poisson geometry

Husileng Xiao

In this talk we will discuss formal super Darbox-Weinstein theorems and then use its equivariant quantum version to give a super Poisson geometric realization of finite W super algebras. Then we will use it to discuss representations of finite W super algebras. The main part of the talk is based on joint work (arXiv:1806.03566) with Bin Shu.