DEGREES OF MODULES AND VARIETIES OF ELEMENTARY ABELIAN LIE ALGEBRAS

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Let $(\mathfrak{g}, [p])$ be a restricted Lie algebra over an algebraically closed field k. Work by Suslin-Friedlander-Bendel shows that the maximal ideal spectrum of the even cohomology ring $\mathrm{H}^{\bullet}(U_0(\mathfrak{g}), k)$ of the restricted universal enveloping algebra $U_0(\mathfrak{g})$ is homeomorphic to the nullcone $V(\mathfrak{g}) := \{x \in \mathfrak{g} \ ; \ x^{[p]} = 0\}$ of $\mathfrak{g}$. In a series of articles, Carlson, Friedlander, Pevtsova and Suslin have introduced full subcategories of $U_0(\mathfrak{g})$-modules, whose objects M are determined by properties of the p-nilpotent operators

$$x^j_M : M \longrightarrow M \ \ ; \ \ m \mapsto x^j.m \qquad (x \in V(\mathfrak{g}), \ j \in \{1, \ldots, p{-}1\}).$$

Given $j \in \{1, \ldots, p{-}1\}$, a $U_0(\mathfrak{g})$-module M is said to have *constant j-rank*, provided

$$\mathrm{rk}(x^j_M) = \mathrm{rk}(y^j_M) \qquad \text{for all } x, y \in V(\mathfrak{g}) \smallsetminus \{0\}.$$

These modules give rise to morphisms

$$\mathsf{im}^j_M : \mathbb{P}(V(\mathfrak{g})) \longrightarrow \mathrm{Gr}_{d_j}(M) \ \ ; \ \ [x] \mapsto \mathrm{im}\, x^j_M$$

from the projectivized nullcone into a Grassmannian. If $\mathfrak{g} = \mathfrak{e}_r$ is an elementary abelian Lie algebra of dimension $r \geq 2$, the above maps induce morphisms $\mathbb{P}^{r-1} \longrightarrow \mathbb{P}^n$. In this talk, which is partly based on joint work with Hao Chang, we study the interplay between M and im^j_M and show how the morphisms im^j_M may be used to define new invariants for such $U_0(\mathfrak{g})$-modules. The observation that these invariants are determined by restricting modules to elementary abelian subalgebras of dimension 2 motivates the investigation of the closed subset

$$\mathbb{E}(2, \mathfrak{g}) := \{\mathfrak{e} \in \mathrm{Gr}_2(\mathfrak{g}) \ ; \ \mathfrak{e} \text{ is an elementary abelian } p\text{-subalgebra}\}$$

of the Grassmannian, which, in contrast to $\mathbb{E}(1, \mathfrak{g}) \cong \mathbb{P}(V(\mathfrak{g}))$, may not be connected.