Tracial matricial structure and approximation by invertibles

Xuanlong Fu

(Fields Institute)

Abstract

If every element in a unital C^*-algebra can be approximated in norm by invertible elements, then such C^*-algebra is called has stable rank one. A non-unital C^*-algebra is called has stable rank one if its minimal unitization has stable rank one. Stable rank one is a frequently occur phenomenon. In 2004, M. Rordam showed that every simple unital finite $\mathcal{Z}$-stable C^*-algebra has stable rank one. In 2015, L. Robert showed that $\mathcal{Z}$-stable projectionless C^*-algebras almost have stable rank one. In a joint work with Kang Li and Huaxin Lin in 2021, we showed that every simple finite $\mathcal{Z}$-stable (not necessary unital) C^*-algebra has stable rank one. And many not necessary $\mathcal{Z}$-stable C^*-algebras (for example, Villadson's first type algebras, and simple unital diagonal AH-algebras) also have stable rank one.

In this talk, I will talk about a joint work with Huaxin Lin in 2021. We showed that for a separable simple C^*-algebra A which has comparison, A has stable rank one is equivalent to has a tracial matricial structure, also equivalent to has tracial approximately oscillation zero, and also equivalent to A has almost stable rank one with the map $\Gamma : Cu(A) \to LAff_+(\tilde{Q}T(A))$ is surjective.