华东师范大学期末试卷

2014－2015 学年第一学期

课程名称: Advanced ODE (International Students)

学生姓名: (Student Name) ________________ 学号(Student ID): ________________

专　　业: Applied Math　　年级/班级: Grade One

课程性质：Required Course (Core) (Closed Exam at 2015/01/13)

一	二	三	四	总分	阅卷人签名

Question 1 (25 Marks) Consider the system: $x' = f(t,x)$, where $f(t,x)$ be continuous and locally Lipschitz on $R^+ \times R^n$. Suppose that there exists an auxiliary function $V(t,x): R^+ \times R^n \to R$ of class C^1 such that

$$W_1(x) \le V(t,x) \le W_2(x),$$

where $W_1(x)$ is semi-positively definite, i.e. $W_1(x) \ge 0$ with $W_1(x) = 0 \Rightarrow x = 0$; and

$$\lim_{\|x\|\to\infty} W_1(x) = \infty;$$

$$V'(t,x) \overset{def\cdot}{=} \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} \cdot f(t,x) \le a + bW_1(x).$$

Show the following:

1) The system has a global solution ($I^+_{\max} = [t_0, \infty)$) for any $(t_0, x_0) \in R^+ \times R^n$.

2) If $W_1(x)$ is positively definite; $a = 0$ and $b < 0$, then, to show that the system is globally uniformly asymptotically stable in Lyapunov sense

Question 2 (25 Marks) Find the bound of solution on $[0, +\infty)$ for the IVP:

$$x' = f(x) = -(1 + x^2)x, \quad x(0) = a,$$

without solving the equation

Question 3 (25 Marks) Show that

$$\dot{x} = A(t)x + h(t)$$

has only $n+1$ linearly independent solutions, where $h(t)$ is not identically zero on I ; $A(t)$ and $h(t)$ are both continuous on I .

Question 4 (25 Marks) Apply LaSalle invariance principle to the following general pendulum equation

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -g(x_1) - h(x_2) \end{cases},$$

where $g(\cdot)$ and $h(\cdot)$ are locally Lipschitz and satisfy

$$g(0) = 0, \quad yg(y) > 0, \quad \forall\, y \neq 0, \quad y \in (-\infty, \infty);$$

$$h(0) = 0, \quad yh(y) > 0, \quad \forall\, y \neq 0, \quad y \in (-\infty, \infty),$$

to conclude its stability of the origin. To guarantee the global result, what additional condition should be imposed?