## MATHEMATICAL SYMBOLS

## Relational Symbols

```
= equal to
 identically equal to
:= equal to by definition
\lll m u c h ~ l e s s ~ t h a n ~
partial order relation
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$\approx$ approximately equal to
$<$ less than
$>$ greater than
$\gg$ much greater than
$\succ$ partial order relation
$\leq$ less than or equal to
$\geq$ greater than or equal to
$\neq$ unequal to, different from
$\triangleq$ corresponding to

## Greek Alphabet

| $A \alpha$ | Alpha | $B \beta$ | Beta | $\Gamma \gamma$ | Gamma | $\Delta \delta$ | Delta | $E \varepsilon$ | Epsilon | $Z \zeta$ | Zeta |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $H \eta$ | Eta | $\Theta \theta \vartheta$ | Theta | $I \iota$ | Iota | $K \kappa$ | Kappa | $A \lambda$ | Lambda | $M \mu$ | Mu |
| $N \nu$ | Nu | $\Xi \xi$ | Xi | $O o$ | Omicron | $\Pi \pi$ | Pi | $P \rho$ | Rho | $\Sigma \sigma$ | Sigma |
| $T \tau$ | Tau | $\gamma v$ | Xpsilon | $\Phi \varphi$ | Phi | $X \chi$ | Chi | $\Psi \psi$ | Psi | $\Omega \omega$ | Omega |

## Constants

| const | constant amount (constant) | $C=0.57722 \ldots$ | Euler constant |
| :--- | :--- | :--- | :--- |
| $\pi=3.14159 \ldots$ | ratio of the perimeter of the circle to | $e=2.71828 \ldots$ | base of the natural logarithms |

Algebra
$A, B \quad$ propositions
$\neg A, \bar{A} \quad$ negation of the proposition $A$
$A \wedge B, \sqcap \quad$ conjunction, logical AND
$A \vee B, \sqcup \quad$ disjunction, logical OR
$A \Rightarrow B \quad$ implication, IF $A$ THEN $B$
$A \Leftrightarrow B \quad$ equivalence, $A$ IF AND ONLY IF $B$

| A, B, C, . | sets | N | set of natural numbers |
| :---: | :---: | :---: | :---: |
| $\bar{A}$ | closure of the set $A$ or complement of | Z | set of the integers |
|  | $A$ with respect to a universal set | Q | set of the rational numbers |
| $A \subset B$ | $A$ is a proper subset of $B$ | $\mathbb{R}$ | set of the real numbers |
| $A \subseteq B$ | $A$ is a subset of $B$ | $\mathbb{R}_{+}$ | set of the positive real numbers |
| $A \backslash B$ | difference of two sets | $\mathbb{R}^{n}$ | $n$-dimensional Euclidean vector space |
| $A \triangle B$ | symmetric difference | $\mathbb{C}$ | set of the complex numbers |
| $A \times B$ | Cartesian product | $R \circ S$ | relation product |
| $x \in A$ | $x$ is an element of $A$ | $x \notin A$ | $x$ is not an element of $A$ |
| $\operatorname{card} A$ | cardinal number of the set $A$ | $\emptyset$ | empty set, zero set |
| $A \cap B$ | intersection of two sets | $\bigcap_{i=1}^{n} A_{i}$ | intersection of $n$ sets $A_{i}$ |
| $A \cup B$ | union of two sets | $\bigcup_{i=1}^{n} A_{i}$ | union of $n$ sets $A_{i}$ |
| $\forall x$ | for all elements $x$ | $\exists x$ | there exists an element $x$ |
| $\{x \in X: p(x)\}$ | subset of all $x$ from $X$ <br> of the property $p(x)$ | $\begin{aligned} & \{x: p(x)\} \\ & \{x \mid p(x)\} \end{aligned}$ | set of all $x$ with the <br> property $p(x)$ |
| $T: X \longrightarrow Y$ | mapping $T$ from the space $X$ into the space $Y$ | $\begin{aligned} & \cong \\ & \sim_{R} \end{aligned}$ | isomorphy of groups equivalence relation |
| $\oplus$ | residue class addition | $\odot$ | residue class multiplication |
| $\begin{aligned} & H=H_{1} \oplus H_{2} \\ & \text { supp } \end{aligned}$ | orthogonal decomposition of space $H$ support | $\mathbf{A} \otimes \mathbf{B}$ | Kronecker product |
| $\sup M$ | supremum: least upper bound of the n | n-empty set | $M(M \subset \mathbb{R})$ bounded above |
| $\inf M$ | infimum: greatest lower bound of the | --empty | $(M \subset \mathbf{R})$ bounded below |


| $[a, b]$ | closed interval, i.e., | $\{x \in \mathbf{R}: a \leq x \leq b\}$ |
| :--- | :--- | :--- |
| $(a, b),] a, b[$ | open interval, i.e., | $\{x \in \mathbf{R}: a<x<b\}$ |
| $(a, b],] a, b]$ | interval open from left, i.e, | $\{x \in \mathbf{R}: a<x \leq b\}$ |
| $[a, b),[a, b[$ | interval open from right, i.e., | $\{x \in \mathbf{R}: a \leq x<b\}$ |

$\operatorname{sign} a \quad \operatorname{sign}$ of the number $a$, e.g., $\operatorname{sign}( \pm 3)= \pm 1, \operatorname{sign} 0=0$
$|a| \quad$ absolute value of the number $a$
$a^{m} \quad a$ to the power $m, a$ to the $m$-th
$\sqrt{a} \quad$ square root of $a$
$\sqrt[n]{a} \quad n$-th root of $a$
$\log _{b} a \quad \operatorname{logarithm}$ of the number $a$ to the base $b$, e.g., $\log _{2} 32=5$
$\log a \quad$ decimal $\log$ arithm (base 10) of the number $a$, e.g., $\lg 100=2$
$\ln a \quad$ natural $\operatorname{logarithm}$ (base $e$ ) of the number $a$, e.g., $\ln e=1$

```
\(a \mid b\)
\(a \nmid b\)
\(a \equiv b \bmod m, a \equiv b(m)\)
g.c.d. \(\left(a_{1}, a_{2}, \ldots, a_{n}\right)\)
l.c.m. \(\left(a_{1}, a_{2}, \ldots, a_{n}\right)\)
\(\binom{n}{k}\)
\(\left(\frac{a}{b}\right)\)
\(n!=1 \cdot 2 \cdot 3 \cdot \ldots \cdot n\)
\((2 n)!!=2 \cdot 4 \cdot 6 \cdot \ldots \cdot(2 n)=2^{n} \cdot n!\);
\((2 n+1)!!=1 \cdot 3 \cdot 5 \cdot \ldots \cdot(2 n+1)\)
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$a$ is a divisor of $b, a$ devides $b$, the ratio of $a$ to $b$
$a$ is not a divisor of $b$
factorial, e.g., $6!=1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6=720 ; \quad$ specially: $0!=1!=1$
in particular: $0!!=1!!=1$
$\mathbf{A}=\left(a_{i j}\right) \quad$ matrix $A$ with elements $a_{i j}$
$\mathbf{A}^{\mathrm{T}} \quad$ transposed matrix
$\mathbf{A}^{-1} \quad$ inverse matrix
$\operatorname{det} \mathbf{A}, \mathrm{D} \quad$ determinant of the square matrix $A$
$\mathbf{E}=\left(\delta_{i j}\right) \quad$ unit matrix
0
$\delta_{i j}$
zero matrix
Kronecker symbol: $\delta_{i j}=0$ for $i \neq j$ and $\delta_{i j}=1$ for $i=j$
a column vector in $\mathbf{R}^{n}$
$\stackrel{\mathbf{a}^{0}}{\underline{a}}$
unit vector in the direction of (parallel to) a
|| $\underline{\mathbf{a}} \|$
$\overrightarrow{\mathbf{a}}, \overrightarrow{\mathbf{b}}, \overrightarrow{\mathbf{c}} \quad$ vectors in $\mathbf{R}^{3}$
norm of $\mathbf{a}$
$\overrightarrow{\mathbf{i}}, \overrightarrow{\mathbf{j}}, \overrightarrow{\mathbf{k}} \quad \overrightarrow{\mathbf{e}}_{x}, \overrightarrow{\mathbf{e}}_{y}, \overrightarrow{\mathbf{e}}_{z} \quad$ basis vectors (orthonormed) of the Cartesian coordinate system
$a_{x}, a_{y}, a_{z} \quad$ coordinates (components) of the vector $\overrightarrow{\mathbf{a}}$
$|\overrightarrow{\mathbf{a}}| \quad$ absolute value, length of the vector $\overrightarrow{\mathbf{a}}$
$\alpha$ a
multiplication of a vector by a scalar
$\overrightarrow{\mathbf{a}} \cdot \overrightarrow{\mathbf{b}}, \overrightarrow{\mathbf{a}} \overrightarrow{\mathbf{b}},(\overrightarrow{\mathbf{a}} \overrightarrow{\mathbf{b}}) \quad$ scalar product, dot product
$\overrightarrow{\mathbf{a}} \times \overrightarrow{\mathbf{b}},[\overrightarrow{\mathbf{a} \mathbf{b}}] \quad$ vector product, cross product
$\overrightarrow{\mathbf{a}} \overrightarrow{\mathbf{b}} \overrightarrow{\mathbf{c}}=\overrightarrow{\mathbf{a}} \cdot(\overrightarrow{\mathbf{b}} \times \overrightarrow{\mathbf{c}}) \quad$ parallelepipedal product, mixed product (triple scalar product)
ㅇ, $\overrightarrow{0}$
zero vector
$\boldsymbol{T}$ tensor
$G=(V, E) \quad$ graph with the set of vertices $V$ and the set of edges $E$

## Geometry



## Complex Numbers

| $\mathrm{i}($ sometimes j$)$ | imaginary unit $\left(\mathrm{i}^{2}=-1\right)$ | $I$ | imaginary unit in computer algebra |
| :--- | :--- | :--- | :--- |
| $\operatorname{Re}(z)$ | real part of the number $z$ | $\operatorname{Im}(z)$ | imaginary part of the number $z$ |
| $\|z\|$ | absolute value of $z$ | $\arg z$ | argument of the number $z$ |
| $\bar{z}$ or $z^{*}$ | complex conjugate of $z$, e.g., $z=2+3 \mathrm{i}$, | $\operatorname{Ln} z$ | logarithm (natural) of a complex num- |
|  | $\bar{z}=2-3 \mathrm{i}$ |  | ber $z$ |

## Trigonometric Functions, Hyperbolic Functions

| $\sin$ | sine | $\cos$ | cosine |
| :---: | :---: | :---: | :---: |
| $\tan$ | tangent | cot | cotangent |
| sec | secant | cosec | cosecant |
| $\arcsin$ | principal value of arc sine (sine inverse) | arccos | principal value of arc cosine (cosine inverse) |
| arctan | principal value of arc tangent (tangent inverse) | arccot | principal value of arc cotangent (cotangent inverse) |
| arcsec | principal value of arc secant (secant inverse) | $\operatorname{arccosec}$ | principal value of arc cosecant (cosecant inverse) |
| sinh | hyperbolic sine | cosh | hyperbolic cosine |
| tanh | hyper bolic tangent | coth | hyperbolic cotangent |
| sech | hyperbolic secant | cosech | hyperbolic cosecant |
| Arsinh | area-hyperbolic sine | Arcosh | area-hyperbolic cosine |
| Artanh | area-hyperbolic tangent | Arcoth | area-hyperbolic cotangent |
| Arsech | area-hyperbolic secant | Arcosech | area-hyperbolic cosecant |

## Analysis

$\lim _{n \rightarrow \infty} x_{n}=A$
$\lim _{x \rightarrow a} f(x)=B$
$f=o(g)$ for $x \rightarrow a$
$f=O(g)$ for $x \rightarrow a$
$\sum_{i=1}^{n}, \sum_{i=1}^{n}$
$\prod_{i=1}^{n}, \prod_{i=1}^{n}$
$f(), \varphi()$
$\Delta$
$d$
$\frac{d}{d x}, \frac{d^{2}}{d x^{2}}, \ldots, \frac{d^{n}}{d x^{n}}$
$\left.\begin{array}{l}f^{\prime}(x), f^{\prime \prime}(x), f^{\prime \prime \prime}(x), \\ f^{(4)}(x), \ldots, f^{(n)}(x) \\ \text { or } \\ \dot{y}, \ddot{y}, \ldots, y^{(n)}\end{array}\right\}$
$A$ is the limit of the sequence $\left(x_{n}\right)$. We also write $x_{n} \rightarrow A$ as $n \rightarrow \infty$;
e.g., $\lim _{n \rightarrow \infty}\left(1+\frac{1}{n}\right)^{n}=e$
$B$ is the limit of the function $f(x)$ as $x$ tends to $a$
Landau symbol "small o" means: $f(x) / g(x) \rightarrow 0$ as $x \rightarrow a$
Landau symbol "big O" means: $f(x) / g(x) \rightarrow C(C=$ const, $C \neq 0)$ as $x \rightarrow a$ sum of $n$ terms for $i$ equals 1 to $n$
product of $n$ factors for $i$ equals 1 to $n$
notation for a function, e.g., $y=f(x), u=\varphi(x, y, z)$
difference or increment, e.g., $\Delta x$ (delta $x$ )
differential, e.g., $d x$ (differential of $x$ )
determination of the first, second, $\ldots, n$-th derivative with respect to $x$
first, second, ..., $n$-th derivative of the function $f(x)$ or of the function $y$
$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial^{2}}{\partial x^{2}}, \ldots$
$\frac{\partial^{2}}{\partial x \partial y}$
$f_{x}, f_{y}, f_{x x}, f_{x y}, f_{y y}, \ldots$
D
grad
div
rot
$\nabla=\frac{\partial}{\partial x} \overrightarrow{\mathbf{i}}+\frac{\partial}{\partial y} \overrightarrow{\mathbf{j}}+\frac{\partial}{\partial z} \overrightarrow{\mathbf{k}}$
$\Delta=\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}+\frac{\partial^{2}}{\partial z^{2}}$
$\frac{\partial \varphi}{\partial \overrightarrow{\mathbf{a}}}$
$\int_{a}^{b} f(x) d x$
$\int_{(C)} f(x, y, z) d s$

$$
\oint_{(C)} f(x, y, z) d s
$$

$$
\iint_{(S)} f(x, y) d S=\iint_{(S)} f(x, y) d x d y
$$

$$
\int_{(S)} f(x, y, z) d S=\iint_{(S)} f(x, y, z) d S
$$

$$
\int_{(V)} f(x, y, z) d V=\iiint_{(V)} f(x, y, z) d x d y d z
$$

$$
\left.\begin{array}{l}
\oint_{(S)} U(\overrightarrow{\mathbf{r}}) \overrightarrow{\mathbf{d} \mathbf{S}}=\oiint_{(S)} U(\overrightarrow{\mathbf{r}}) \overrightarrow{\mathbf{d S}} \\
\oint_{(S)} \overrightarrow{\mathbf{V}}(\overrightarrow{\mathbf{r}}) \cdot \overrightarrow{\mathbf{d S}}=\oiint_{(S)} \overrightarrow{\mathbf{V}}(\overrightarrow{\mathbf{r}}) \cdot \overrightarrow{\mathbf{d S}} \\
\oint_{(S)} \overrightarrow{\mathbf{V}}(\overrightarrow{\mathbf{r}}) \times \overrightarrow{\mathbf{d} \mathbf{S}}=\oiint_{(S)} \overrightarrow{\mathbf{V}}(\overrightarrow{\mathbf{r}}) \times \overrightarrow{\mathbf{d S}}
\end{array}\right\}
$$

$A=\max !$
$A=\max$
determination of the first, second, ..., $n$-th partial derivative
determination of the second partial derivative first with respect to $x$, then with respect to $y$
first, second, ... partial derivative of function $f(x, y)$
differential operator, e.g., $D y=y^{\prime}, D^{2} y=y^{\prime \prime}$
gradient of a scalar field $(\operatorname{grad} \varphi=\nabla \varphi)$
divergence of a vector field ( $\operatorname{div} \overrightarrow{\mathbf{v}}=\nabla \cdot \overrightarrow{\mathbf{v}})$
rotation or curl of a vector field (rot $\overrightarrow{\mathbf{v}}=\nabla \times \overrightarrow{\mathbf{v}})$
nabla operator, here in Cartesian coordinates (also called the Hamiltonian differential operator, not to be confused with the Hamilton operator in quantum mechanics)

Laplace operator
directional derivative, i.e., derivative of a
scalar field $\varphi$ into the direction $\overrightarrow{\mathbf{a}}: \frac{\partial \varphi}{\partial \overrightarrow{\mathbf{a}}}=\overrightarrow{\mathbf{a}} \cdot \operatorname{grad} \varphi$
definite integral of the function $f$ between the limits $a$ and $b$
line integral of the first kind with respect to the space curve $C$ with arclength $s$
integral along a closed curve (circulatory integral)
double integral over a planar region $S$
surface integral of the first kind over a spatial surface $S$ (see (8.152b), p. 482)
triple integral or volume integral over the volume $V$
surface integrals over a closed surface in vector analysis
expression $A$ is to be maximized, similarly min!, extreme!
expression $A$ is maximal, similarly min, extreme.

