A new q-analogue of Van Hamme's (G.2) supercongruence for primes $p \equiv 3 \pmod 4$

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Abstract. Van Hamme's (G.2) supercongruence modulo p^4 for primes $p \equiv 3 \pmod 4$ and $p > 3$ was first established by Swisher. A q-analogue of this supercongruence was implicitly given by the author and Schlosser. In this paper, we present a new q-analogue of Van Hamme's (G.2) supercongruence for $p \equiv 3 \pmod 4$.

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1. Introduction

In his first letter to Hardy in 1913, Ramanujan asserted that (see [2, p. 25, eq. (2)]):

$$\sum_{k=0}^{\infty}(8k+1)\frac{(\frac{1}{4})_k^4}{k!^4}=\frac{2\sqrt{2}}{\sqrt{\pi}\,\Gamma(\frac{3}{4})^2} \tag{1}$$

without proof. Here $(a)_n = a(a+1)\cdots(a+n-1)$ is the Pochhammer symbol and $\Gamma(x)$ is the Gamma function. The formula (1) was later proved by Hardy [10]. In 1997, Van Hamme [9] listed thirteen p-adic analogues of Ramanujan-type series, such as: for $p \equiv 1 \pmod 4$,

$$\sum_{k=0}^{(p-1)/4}(8k+1)\frac{(\frac{1}{4})_k^4}{k!^4}\equiv p\frac{\Gamma_p(\frac{1}{2})\Gamma_p(\frac{1}{4})}{\Gamma_p(\frac{3}{4})} \pmod{p^3} \tag{2}$$

(tagged (G.2) in Van Hamme's list). Here and in what follows, p is an odd prime and $\Gamma_p(x)$ denotes the p-adic Gamma function [19]. Swisher [20] and He [11] proved that (2) is true modulo the higher power p^4. Swisher [20, (3)] also proved the following generalization of Van Hamme's (G.2) supercongruence: for $p \equiv 3 \pmod 4$ and $p > 3$,

$$\sum_{k=0}^{(3p-1)/4}(8k+1)\frac{(\frac{1}{4})_k^4}{k!^4}\equiv -\frac{3p^2\Gamma_p(\frac{1}{2})\Gamma_p(\frac{1}{4})}{2\Gamma_p(\frac{3}{4})} \pmod{p^4} \tag{3}$$

(The factor (-1) was neglected by Swisher in her original supercongruence).

In the past few years, q-analogues of Van Hamme's supercongruences have been widely studied. For example, the author and Schlosser [5, Corollary 1.2 with $d=4$] gave the following q-analogue of (3): for $n\equiv 3 \pmod 4$,

$$\sum_{k=0}^{M}[8k+1]\frac{(q;q^4)_k^4}{(q^4;q^4)_k^4}q^{2k}\equiv\frac{(q^2;q^4)_{(3n-1)/4}}{(q^4;q^4)_{(3n-1)/4}}[3n]q^{(1-3n)/4}\pmod{[n]\Phi_n(q)^3},\tag{4}$$

where $M=(3n-1)/4$ or $n-1$. Here, the q-*shifted factorial* is defined by $(a;q)_0=1$ and $(a;q)_n=(1-a)(1-aq)\cdots(1-aq^{n-1})$ for $n=1,2,\dots$. For convenience, we also adopt the abbreviated notation $(a_1,a_2,\dots,a_m;q)_n=(a_1;q)_n(a_2;q)_n\cdots(a_m;q)_n$. Moreover, the q-*integer* is defined as $[n]=[n]_q=(1-q^n)/(1-q)$, and $\Phi_n(q)$ denotes the n-th *cyclotomic polynomial*, i.e.,

$$\Phi_n(q)=\prod_{\substack{1\leqslant k\leqslant n\\ \gcd(k,n)=1}}(q-\zeta^k),$$

where ζ is an n-th primitive root of unity.

Liu and Wang [15] showed that Van Hamme's original (G.2) supercongruence can be deduced from the following q-supercongruence: for $n\equiv 1 \pmod 4$,

$$\sum_{k=0}^{M}[8k+1]\frac{(q;q^4)_k^4}{(q^4;q^4)_k^4}q^{2k}\equiv\frac{(q^2;q^4)_{(n-1)/4}}{(q^4;q^4)_{(n-1)/4}}[n]q^{(1-n)/4}\pmod{[n]\Phi_n(q)^2},\tag{5}$$

where $M=(n-1)/4$ or $n-1$. Very recently, Liu and Wang [17] gave a generalization of (5) modulo $[n]\Phi_n(q)^3$. For another generalization of (5), see [5, Theorem 4.3]. Liu and Wang [15] also established the following q-supercongruence: for $n\equiv 1 \pmod 4$, modulo $[n]_{q^2}\Phi_n(q^2)^2$,

$$\sum_{k=0}^{M}[8k+1]_{q^2}[8k+1]^2\frac{(q^2;q^8)_k^4}{(q^8;q^8)_k^4}q^{-4k}\equiv-\frac{2(q^4;q^8)_{(n-1)/4}}{(1+q^2)(q^8;q^8)_{(n-1)/4}}[n]_{q^2}q^{(3-n)/2},\tag{6}$$

where $M=(n-1)/4$ or $n-1$.

It is easy to see that the $n=p$ and $q\to -1$ case of (6) reduces to (2). Moreover, letting $n=p$ and $q\to 1$ in (6), Liu and Wang obtained the following new supercongruence: for $p\equiv 1 \pmod 4$,

$$\sum_{k=0}^{(p-1)/4}(8k+1)^3\frac{(\frac14)_k^4}{k!^4}\equiv -p\frac{\Gamma_p(\frac12)\Gamma_p(\frac14)}{\Gamma_p(\frac34)}\pmod{p^3}.$$

In this paper, we shall establish the following new q-analogue of (3).

Theorem 1.1. *Let $n \equiv 3 \pmod 4$ be a positive integer. Then, modulo $[n]_{q^2}\Phi_n(q^2)^3$,*

$$\sum_{k=0}^{M}[8k+1]_{q^2}[8k+1]^2\frac{(q^2;q^8)_k^4}{(q^8;q^8)_k^4}q^{-4k} \equiv -\frac{2(q^4;q^8)_{(3n-1)/4}}{(1+q^2)(q^8;q^8)_{(3n-1)/4}}[3n]_{q^2}q^{(3-3n)/2}, \qquad (7)$$

where $M = (3n-1)/4$ or $n-1$.

For some other recent work on q-supercongruences, see [1, 6–8, 12–14, 16, 21, 22].

To see that the q-supercongruences (4) and (7) are indeed q-analogues of (3), we need to prove the following result.

Proposition 1.2. *Let $p \equiv 3 \pmod 4$ and $p > 3$. Then*

$$\frac{(\frac{1}{2})_{(3p-1)/4}}{(1)_{(3p-1)/4}} \equiv -\frac{p\Gamma_p(\frac{1}{2})\Gamma_p(\frac{1}{4})}{2\Gamma_p(\frac{3}{4})} \pmod{p^3}. \qquad (8)$$

It is easy to see that the $n = p$ and $q \to -1$ case of (7) reduces to (3). Meanwhile, taking $n = p$ and $q \to 1$ in (7), we get the following new result: for $p \equiv 3 \pmod 4$ and $p > 3$,

$$\sum_{k=0}^{(3p-1)/4}(8k+1)^3\frac{(\frac{1}{4})_k^4}{k!^4} \equiv \frac{3p^2\Gamma_p(\frac{1}{2})\Gamma_p(\frac{1}{4})}{2\Gamma_p(\frac{3}{4})} \pmod{p^4}.$$

We shall prove Theorem 1.1 in the next section employing the method of 'creative microscoping', introduced by the author and Zudilin [7]. A simple proof of Proposition 1.2 using properties of the p-adic Gamma function will be given in Section 3.

2. Proof of Theorem 1.1

We will make use of Watson's ${}_8\phi_7$ transformation formula (see [3, Appendix (III.18)]):

$$\begin{aligned}&{}_8\phi_7\left[\begin{matrix}a, & qa^{\frac{1}{2}}, & -qa^{\frac{1}{2}}, & b, & c, & d, & e, & q^{-n} \\ & a^{\frac{1}{2}}, & -a^{\frac{1}{2}}, & aq/b, & aq/c, & aq/d, & aq/e, & aq^{n+1}\end{matrix};q,\ \frac{a^2q^{n+2}}{bcde}\right] \\ &\quad = \frac{(aq, aq/de;q)_n}{(aq/d, aq/e;q)_n}\,{}_4\phi_3\left[\begin{matrix}aq/bc,\ d,\ e,\ q^{-n} \\ aq/b,\ aq/c,\ deq^{-n}/a\end{matrix};q,\ q\right], \qquad (9)\end{aligned}$$

where the *basic hypergeometric series* ${}_{r+1}\phi_r$ is defined as

$${}_{r+1}\phi_r\left[\begin{matrix}a_1, a_2, \ldots, a_{r+1} \\ b_1, b_2, \ldots, b_r\end{matrix};q,\ z\right] = \sum_{k=0}^{\infty}\frac{(a_1, a_r\cdots, a_{r+1};q)_k}{(q, b_1, \ldots, b_r;q)_k}z^k.$$

We shall also utilize the following easily proved q-congruence due to the author and Schlosser [4, Lemma 3].

Lemma 2.1. *Let d, m and n be positive integers with $m \leqslant n-1$ and $dm \equiv -1 \pmod n$. Then, for $0 \leqslant k \leqslant m$, we have*

$$\frac{(aq;q^d)_{m-k}}{(q^d/a;q^d)_{m-k}} \equiv (-a)^{m-2k} \frac{(aq;q^d)_k}{(q^d/a;q^d)_k} q^{m(dm-d+2)/2+(d-1)k} \pmod{\Phi_n(q)}.$$

We first present the following q-congruence with two parameters a and b.

Theorem 2.2. *Let $n \equiv 3 \pmod 4$ be a positive integer, and let a, b be indeterminates. Then, modulo $\Phi_n(q^2)(1-aq^{6n})(a-q^{6n})$,*

$$\sum_{k=0}^{(3n-1)/4} [8k+1]_{q^2}[8k+1]^2 \frac{(aq^2,q^2/a,q^2/b,q^2;q^8)_k}{(aq^8,q^8/a,bq^8,q^8;q^8)_k}\left(\frac{b}{q^4}\right)^k$$
$$\equiv b^{(3n-1)/4}q^{(1-3n)/2}[3n]_{q^2}\frac{(q^4/b;q^8)_{(3n-1)/4}}{(bq^8;q^8)_{(3n-1)/4}}\left(1-\frac{(1-aq^2)(1-q^2/a)(1-q^2/b)}{(1-q)^2(1-q^4/b)}\right). \quad (10)$$

Proof. For $a=q^{-6n}$ or $a=q^{6n}$, the left-hand side of (10) is equal to

$$\sum_{k=0}^{(3n-1)/4} [8k+1]_{q^2}[8k+1]^2 \frac{(q^{2-6n},q^{2+6n},q^2/b,q^2;q^8)_k}{(q^{8-6n},q^{8+6n},bq^8,q^8;q^8)_k} b^k q^{-4k}$$
$$= {}_8\phi_7\left[\begin{matrix} q^2, & q^9, & -q^9, & q^9, & q^9, & q^2/b, & q^{2+6n}, & q^{2-6n} \\ & q, & -q, & q, & q, & bq^8, & q^{8-6n}, & q^{8+6n} \end{matrix}; q^8,\, bq^{-4}\right]. \quad (11)$$

By Watson's ${}_8\phi_7$ transformation formula (9), the right-hand side of (11) can be written as

$$\frac{(q^{10},bq^{6-6n};q^8)_{(3n-1)/4}}{(bq^8,q^{8-6n};q^8)_{(3n-1)/4}}\, {}_4\phi_3\left[\begin{matrix} q^{-8}, & q^2/b, & q^{2+2n}, & q^{2-2n} \\ & q, & q, & q^4/b \end{matrix}; q^8,\, q^8\right]$$
$$= b^{(3n-1)/4}q^{(1-3n)/2}[3n]_{q^2}\frac{(q^4/b;q^8)_{(3n-1)/4}}{(bq^8;q^8)_{(3n-1)/4}}\left(1-\frac{(1-q^{2-2n})(1-q^{2+2n})(1-q^2/b)}{(1-q)^2(1-q^4/b)}\right). \quad (12)$$

This means that (10) holds modulo $1-aq^{6n}$ and $a-q^{6n}$.

Moreover, setting $q \mapsto q^2$, $d=4$, and $m=(3n-1)/4$ in Lemma 2.1, for $0 \leqslant k \leqslant m$, we have

$$\frac{(aq^2;q^8)_{m-k}}{(q^8/a;q^8)_{m-k}} \equiv (-a)^{m-2k}\frac{(aq^2;q^8)_k}{(q^8/a;q^8)_k} q^{2m(2m-1)+6k} \pmod{\Phi_n(q^2)}.$$

Using this q-congruence, we can easily verify that the k-th and $((3n-1)/4-k)$-th summands on the left-hand side of (10) modulo $\Phi_n(q^2)$ cancel each other for $0 \leqslant k \leqslant (3n-1)/4$. This proves that the left-hand side of (10) is congruent to 0 modulo $\Phi_n(q^2)$, and so (10) is true modulo $\Phi_n(q^2)$.

The proof then follows from the fact that $1-aq^{6n}$, $a-q^{6n}$, and $\Phi_n(q^2)$ are pairwise coprime polynomials in q. □

We also need a simpler q-congruence as follows.

Theorem 2.3. *Let $n \equiv 3 \pmod 4$ be a positive integer, and let a, b be indeterminates. Then, modulo $b - q^{6n}$,*

$$\sum_{k=0}^{(3n-1)/4} [8k+1]_{q^2}[8k+1]^2 \frac{(aq^2, q^2/a, q^2/b, q^2; q^8)_k}{(aq^8, q^8/a, bq^8, q^8; q^8)_k} \left(\frac{b}{q^4}\right)^k \equiv \frac{[3n]_{q^2}(q^2, q^6; q^8)_{(3n-1)/4}}{(aq^8, q^8/a; q^8)_{(3n-1)/4}} \left(1 - \frac{(1-aq^2)(1-q^2/a)(1-q^2/b)}{(1-q)^2(1-q^4/b)}\right). \qquad (13)$$

Proof. For $b = q^{6n}$, the left-hand side of (13) is equal to

$$\sum_{k=0}^{(3n-1)/4} [8k+1]_{q^2}[8k+1]^2 \frac{(aq^2, q^2/a, q^{2-6n}, q^2; q^8)_k}{(aq^8, q^8/a, q^{8+6n}, q^8; q^8)_k} q^{(6n-4)k} = {}_8\phi_7\left[\begin{matrix} q^2, & q^9, & -q^9, & q^9, & q^9, & aq^2, & q^2/a, & q^{2-6n} \\ & q, & -q, & q, & q, & q^8/a, & aq^8, & q^{8+6n} \end{matrix}; q^8, q^{6n-4}\right]. \qquad (14)$$

In view of Watson's transformation (9), we can write the right-hand side of (14) as

$$\frac{(q^{10}, q^6; q^8)_{(3n-1)/4}}{(aq^8, q^8/a; q^8)_{(3n-1)/4}} {}_4\phi_3\left[\begin{matrix} q^{-8}, & aq^2, & q^2/a, & q^{2-6n} \\ & q, & q, & q^{4-6n} \end{matrix}; q^8, q^8\right] = \frac{[3n]_{q^2}(q^2, q^6; q^8)_{(3n-1)/4}}{(aq^8, q^8/a; q^8)_{(3n-1)/4}} \left(1 - \frac{(1-aq^2)(1-q^2/a)(1-q^{2-6n})}{(1-q)^2(1-q^{4-6n})}\right). \qquad (15)$$

This proves that the congruence (13) is true modulo $b - q^{6n}$. □

We are now able to establish the following parametric generalization of Theorem 1.1.

Theorem 2.4. *Let $n \equiv 3 \pmod 4$ be a positive integer, and let a, b be indeterminates. Then, modulo $\Phi_n(q^2)^2(1-aq^{6n})(a-q^{6n})$,*

$$\sum_{k=0}^{(3n-1)/4} [8k+1]_{q^2}[8k+1]^2 \frac{(aq^2, q^2/a, q^2, q^2; q^8)_k}{(aq^8, q^8/a, q^8, q^8; q^8)_k} q^{-4k} \equiv q^{(1-3n)/2}[3n]_{q^2} \frac{(q^4/b; q^8)_{(3n-1)/4}}{(bq^8; q^8)_{(3n-1)/4}} \left(1 - \frac{(1-aq^2)(1-q^2/a)(1-q^2/b)}{(1-q)^2(1-q^4/b)}\right). \qquad (16)$$

Proof. It is obvious that $\Phi_n(q^2)(1-aq^{6n})(a-q^{6n})$ and $b-q^{6n}$ are relatively prime polynomials. Employing the Chinese reminder theorem for coprime polynomials, we can determine the remainder of the left-hand side of (10) modulo $\Phi_n(q^2)(1-aq^{6n})(a-q^{6n})(b-q^{6n})$ from (10) and (13):

$$\sum_{k=0}^{(3n-1)/4} [8k+1]_{q^2}[8k+1]^2 \frac{(aq^2, q^2/a, q^2/b, q^2; q^8)_k}{(aq^8, q^8/a, bq^8, q^8; q^8)_k} \left(\frac{b}{q^4}\right)^k$$

$$\equiv b^{(3n-1)/4} q^{(1-3n)/2} [3n]_{q^2} \frac{(q^4/b; q^8)_{(3n-1)/4}}{(bq^8; q^8)_{(3n-1)/4}} \left(1 - \frac{(1-aq^2)(1-q^2/a)(1-q^2/b)}{(1-q)^2(1-q^4/b)}\right)$$

$$\times \frac{(b-q^{6n})(ab-1-a^2+aq^{6n})}{(a-b)(1-ab)}$$

$$+ \frac{[3n]_{q^2}(q^2, q^6; q^8)_{(3n-1)/4}}{(aq^8, q^8/a; q^8)_{(3n-1)/4}} \left(1 - \frac{(1-aq^2)(1-q^2/a)(1-q^2/b)}{(1-q)^2(1-q^4/b)}\right)$$

$$\times \frac{(1-aq^{6n})(a-q^{6n})}{(a-b)(1-ab)} \pmod{\Phi_n(q^2)(1-aq^{6n})(a-q^{6n})(b-q^{6n})}. \tag{17}$$

Here we have used the following q-congruences:

$$\frac{(b-q^{6n})(ab-1-a^2+aq^{6n})}{(a-b)(1-ab)} \equiv 1 \pmod{(1-aq^{6n})(a-q^{6n})},$$

$$\frac{(1-aq^{6n})(a-q^{6n})}{(a-b)(1-ab)} \equiv 1 \pmod{b-q^{6n}}.$$

Note that $1-q^{6n}$ contains the factor $\Phi_n(q^2)$ and so do $(q^4;q^8)_{(3n-1)/4}$ and $(q^6;q^8)_{(3n-1)/4}$ since they have the factors $1-q^{4n}$ and $1-q^{2n}$, respectively. Moreover, the factor $(bq^8;q^8)_{(3n-1)/4}$ in the denominators of both sides of (17) is relatively prime to $\Phi_n(q^2)$ when $b=1$. Thus, letting $b=1$ in (17) and observing that

$$(1-q^{6n})(1+a^2-a-aq^{6n}) = (1-a)^2 + (1-aq^{6n})(a-q^{6n}),$$

we see that the right-hand of (17) reduces to

$$q^{(1-3n)/2}[3n]_{q^2} \frac{(q^4/b; q^8)_{(3n-1)/4}}{(bq^8; q^8)_{(3n-1)/4}} \left(1 - \frac{(1-aq^2)(1-q^2/a)(1-q^2/b)}{(1-q)^2(1-q^4/b)}\right)$$
$$\pmod{\Phi_n(q^2)^2(1-aq^{6n})(a-q^{6n})},$$

as desired. □

Proof of Theorem 1.1. Taking $a=1$ in (16), we see that the q-congruence (7) holds modulo $\Phi_n(q^2)^4$ for $M=(3n-1)/4$. It is easy to see that $(q^2;q^8)_k^4/(q^8;q^8)_k^4$ is congruent to 0 modulo $\Phi_n(q^2)^4$ for any k in the range $(3n-1)/4 < k \leqslant n-1$. Therefore, the q-congruence (7) also holds modulo $\Phi_n(q^2)^4$ for $M=n-1$.

Moreover, similarly to the proof of [5, Lemma 2.2], we can prove that (7) holds modulo $[n]_{q^2}$. Since the least common multiple of $[n]_{q^2}$ and $\Phi_n(q^2)^4$ is $[n]_{q^2}\Phi_n(q^2)^3$, we complete the proof of the theorem. □

3. Proof of Proposition 1.2

We first list some basic properties of Morita's p-adic Gamma function. Let p be an odd prime. Set $\Gamma_p(0) = 1$, and for all integers $n \geqslant 1$, the p-adic Gamma function is defined as

$$\Gamma_p(n) = (-1)^n \prod_{\substack{0<k<n \\ p\nmid k}} k.$$

Let $\mathbb{Z}_p$ denote the ring of all p-adic integers. Extend Γ_p to all $x \in \mathbb{Z}_p$ by defining

$$\Gamma_p(x) = \lim_{x_n \to x} \Gamma_p(x_n),$$

where x_n is any sequence of positive integers p-adically approaching x. The following facts can be found in [18]: for any $x \in \mathbb{Z}_p$,

$$\frac{\Gamma_p(x+1)}{\Gamma_p(x)} = \begin{cases} -x, & p \nmid x, \\ -1, & p \mid x. \end{cases} \tag{18}$$

$$\Gamma_p(x)\Gamma_p(1-x) = (-1)^{a_0(x)}, \tag{19}$$

where $a_0(x) \in \{1, 2, \ldots, p\}$ satisfies $a_0(x) \equiv x \pmod p$.

In order to prove Proposition 1.2, we also need the following result (see [18, Theorem 14]).

Lemma 3.1. *For any odd prime p and $a, m \in \mathbb{Z}_p$, we have*

$$\Gamma_p(a + mp) \equiv \Gamma_p(a) + \Gamma_p'(a)mp \pmod{p^2}. \tag{20}$$

Proof of Proposition 1.2. By the properties (18)–(20), for $p \equiv 3 \pmod 4$ and $p > 3$,

$$\begin{aligned} \frac{(\frac{1}{2})_{(3p-1)/4}}{(1)_{(3p-1)/4}} = \frac{p}{2}\frac{\Gamma_p(1)\Gamma_p(\frac{3p+1}{4})}{\Gamma_p(\frac{1}{2})\Gamma_p(\frac{3p+3}{4})} &= (-1)^{(3p+3)/4}\frac{p\Gamma_p(1)\Gamma_p(\frac{3p+1}{4})\Gamma_p(\frac{1-3p}{4})}{2\Gamma_p(\frac{1}{2})} \\ &\equiv (-1)^{(3p+3)/4}\frac{p\Gamma_p(1)\Gamma_p(\frac{1}{4})^2}{2\Gamma_p(\frac{1}{2})} \\ &\equiv \frac{p\Gamma_p(1)\Gamma_p(\frac{1}{4})}{2\Gamma_p(\frac{1}{2})\Gamma_p(\frac{3}{4})} \pmod{p^3}. \end{aligned}$$

Noticing that $\Gamma_p(1) = -1$ and $\Gamma_p(\frac{1}{2})^2 = (-1)^{\frac{p+1}{2}} = 1$, we complete the proof. □

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