

常用矢量运算公式:

$$a \cdot (b \times c) = b \cdot (c \times a) - c \cdot (a \times b)$$

$$a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$$

$$(a \times b) \cdot (c \times d) = (a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c)$$

$$\nabla \times \nabla \phi = 0, \quad \nabla \cdot (\nabla \times a) = 0$$

$$\nabla \times (\nabla \times a) = \nabla (\nabla \cdot a) - \Delta a, \quad \nabla^2 a = \Delta a$$

$$\nabla \cdot (\phi a) = a \cdot \nabla \phi + \phi \nabla \cdot a$$

$$\nabla \times (\phi a) = \nabla \phi \times a + \phi \nabla \times a$$

$$\nabla (a \cdot b) = (a \cdot \nabla)b + (b \cdot \nabla)a + a \times (\nabla \times b) + b \times (\nabla \times a)$$

$$\nabla \cdot (a \times b) = b \cdot (\nabla \times a) - a \cdot (\nabla \times b)$$

$$\nabla \times (a \times b) = a(\nabla \cdot b) - b(\nabla \cdot a) + (b \cdot \nabla)a - (a \cdot \nabla)b$$

$$\mathbf{x} \in \mathbb{R}^3: \quad r = |\mathbf{x}|, \quad n = \frac{\mathbf{x}}{r}$$

$$\nabla \times \mathbf{x} = \mathbf{0}, \quad \nabla \cdot \mathbf{x} = 3, \quad \nabla \cdot [n f(r)] = \frac{2}{r} f + \frac{\partial f}{\partial r}$$

$$\nabla \times [n f(r)] = \mathbf{0},$$

$$(a \cdot \nabla) n f(r) = \frac{f(r)}{r} [a - n(a \cdot n)] + n(a \cdot n) \frac{\partial f}{\partial r},$$

$$\nabla (\mathbf{x} \cdot a) = a + \mathbf{x}(\nabla \cdot a) + i(L \times a), \quad L = \frac{1}{i} (\mathbf{x} \times \nabla) \text{ 为角动量算子.}$$

记号:

$$u = (u^1, \dots, u^n)$$

$$u \otimes u = u^T \cdot u = \begin{pmatrix} u^1 u^1 & \dots & u^1 u^n \\ \vdots & & \vdots \\ u^n u^1 & \dots & u^n u^n \end{pmatrix}$$