P.226.(23) 设M是R^n中$k+l+1$维有向无边嵌入子流形，ω,η分别是定义在R^n中包含M在内的一个开子集上的k次和l次外微分式. 证明：存在某个实数a，使得

$$\int_M \omega\wedge d\eta = a\int_M d\omega\wedge\eta,$$

并求出a.

解：设$(f:M)$为R^n中嵌入子流形，则由 $d(\omega\wedge\eta) = d\omega\wedge\eta + (-1)^k\omega\wedge d\eta$

$$f^*d(\omega\wedge\eta) = d(f^*(\omega\wedge\eta)) = f^*d\omega\wedge f^*\eta + (-1)^k f^*\omega\wedge f^*d\eta$$
$$= d(f^*\omega)\wedge f^*\eta + (-1)^k f^*\omega\wedge df^*\eta.$$

从而

$$(-1)^k\int_M \omega\wedge d\eta = \int_M f^*(\omega\wedge d\eta)\cdot(-1)^k = \int_M d(f^*(\omega\wedge\eta)) - \int_M d(f^*\omega)\wedge f^*\eta$$

$$= \underbrace{\int_{\partial M} f^*(\omega\wedge\eta)}_{0} - \int_M f^*(d\omega\wedge\eta) = -\int_M f^*(d\omega\wedge\eta) = -\int_M d\omega\wedge\eta$$

$\therefore\ a = (-1)^{k+1}$. #.

(24) 设$\omega = A\,dy\wedge dz + B\,dz\wedge dx + C\,dx\wedge dy$是$R^3$中的2次外微分式，且$d\omega=0$. 令

$$\alpha = \int_0^1 tA(tx,ty,tz)dt\cdot(ydz - zdy) + \int_0^1 tB(tx,ty,tz)\,dt\cdot(zdx - xdz)$$
$$+\int_0^1 tC(tx,ty,tz)dt\cdot(xdy - ydx).$$

证明$d\alpha = \omega$.

证：P.193 Poincaré 引理证明的特例. #.