Constructions of Covering Arrays with Strength 3 and 4 from Difference Covering Array

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Contents

1. Introduction

Definition: Covering Array

A covering array CA$(N; t, k, v)$ is an $N \times k$ array with entries from a set X of v symbols such that every $N \times t$ sub-array contains all t-tuples over X at least once, where the parameter t is the strength of the coverage of intersections, k is the number of factors, and v is the number of levels associated with each factor (*order*).

Covering array number CAN(t, k, v): the minimum size N for which a CA$(N; t, k, v)$ exists.

An example

CA(t, k, v) is an $N \times k$ array where every $N \times t$ sub-array contains all t-tuples at least once.

CAN$(2, 4, 2) = 5$

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CA(t, k, v) is an $N \times k$ array where every $N \times t$ sub-array contains all t-tuples at least once.

CAN$(2, 4, 2) = 5$

1	1	1	1
1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Orthogonal array

Definition: Orthogonal Array

An orthogonal array OA(t, k, v) is an $N \times k$ array with entries from a set X of v symbols such that every $N \times t$ sub-array contains all t-tuples over X exactly once.

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Example: OA$(3, 4, 2)$

0	0	0	0
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0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
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Known result on OAs

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2. If $q = 2^r$, then there is an OA$(3, q+2, q)$. (Bush 1952)
3. If OA(t, k, s_i) exist for $1 \le i \le m$, then an OA$(t, k, \prod_{1\le i\le m} s_i)$. (Bush 1952)

Known result on OAs

1. For any prime power q and $t \leq q$, there is an $\mathrm{OA}(t, q+1, q)$. (Bush 1952)
2. If $q = 2^r$, then there is an $\mathrm{OA}(3, q+2, q)$. (Bush 1952)
3. If $\mathrm{OA}(t, k, s_i)$ exist for $1 \leq i \leq m$, then an $\mathrm{OA}(t, k, \prod_{1\leq i\leq m} s_i)$. (Bush 1952)

Difference Covering Array

A difference covering array, or a DCA$(k, n; v)$ is a $k \times n$ array (a_{ij}) $(1 \leq i \leq k,\ 1 \leq j \leq n)$ with entries from an abelian group G of order v, such that, for any two distinct rows l and h of D $(1 \leq l < h \leq k)$, the difference list

$$\Delta_{lh} = \{d_{h1} - d_{l1}, d_{h2} - d_{l2}, \ldots, d_{hn} - d_{ln}\}$$

contains every element of G at least once. When 'at least' is replaced by 'exactly', this defines a difference matrix $(v, k; 1)$-DM.

2. CA(3, 5, v)

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1. For all even positive integers v, there exists a DCA($4, v + 1; v$). (Yin 2005)
2. If $v \not\equiv 2\ (mod\ 4)$ and $v \geq 4$, then there is a $(v, 4; 1)$-DM. (Ge, 2005)

A CA$(nv^2; 3, 5, v)$ via DCA $(4, n; v)$

Construction 1

If there exists a DCA$(4, n; v)$, then there exists a CA$(nv^2; 3, 5, v)$.

A CA$(nv^2; 3, 5, v)$ via DCA $(4, n; v)$

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If there exists a DCA$(4, n; v)$, then there exists a CA$(nv^2; 3, 5, v)$.

Proof. Let $D = (d_{ij})$ be the given DCA$(4, n; v)$ over the abelian group G. A CA$(v^2 n; 3, 5, v)$ on G consists of the following rows:

$$(d_{1j} + u, d_{2j} + u, d_{3j} + u + e, d_{4j} + u + e, e),$$

where $(d_{1j}, d_{2j}, d_{3j}, d_{4j})^T$ is the jth column of the DCA, and $e, u \in G$.

A CA$(nv^2; 3, 5, v)$ via DCA $(4, n; v)$

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where $(d_{1j}, d_{2j}, d_{3j}, d_{4j})^T$ is the jth column of the DCA, and $e, u \in G$.

Theorem 1

For any even positive integers v, CAN$(3, 5, v) \leq v^3 + v^2$.

New OA$(3, 5, v)$

Construction for OA$(3, 5, v)$

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Corollary

If $v \not\equiv 2 \ (mod\ 4)$ and $v \geq 4$, then there is an OA$(3, 5, v)$.

New $\mathrm{OA}(3, 5, v)$

Construction for $\mathrm{OA}(3, 5, v)$

If there is a $(v, 4; 1)$-DM, then there is an $\mathrm{OA}(3, 5, v)$.

Corollary

If $v \not\equiv 2 \ (mod\ 4)$ and $v \geq 4$, then there is an $\mathrm{OA}(3, 5, v)$.

Question:

Is there an $\mathrm{OA}(3, 5, v)$ for $v \equiv 2 \ (mod\ 4)$?

Holey Difference Matrix

Suppose that G is an abelian group of order v that contains a subgroup H of order w. A $k \times (v-w)$ matrix $D = (d_{ij})$ $(1 \leq i \leq k, 1 \leq j \leq v-w)$ over G is termed a holey difference matrix (HDM) of one hole if, for any two distinct rows l and h of D $(1 \leq l < h \leq k)$, the difference list

$$\Delta_{lh} = \{d_{h1} - d_{l1}, d_{h2} - d_{l2}, \ldots, d_{h(v-w)} - d_{l(v-w)}\}$$

contains every element of $G \setminus H$ exactly once, while any element of H does not appear in Δ_{lh} (and hence H is a hole). For convenience, we shall simply refer to such a matrix D as a $(k, v; w)$-HDM over $(G; H)$.

For any odd positive integer v satisfying $gcd(v, 9) \neq 3$, there exists a $(4, 2v; 2)$-HDM. (Yin 2005)

$\mathrm{CA}(3,5,v)$ from $(4,v;w)$-HDM

Construction 2

If there exists a $(4,v;w)$-HDM and a $\mathrm{CA}(b;3,5,w)$, then $\mathrm{CAN}(3,5,v) \leq v^3 - v^2w + b(\frac{v}{w})^2$.

$CA(3,5,v)$ from $(4,v;w)$-HDM

Construction 2

If there exists a $(4,v;w)$-HDM and a $CA(b;3,5,w)$, then $CAN(3,5,v) \leq v^3 - v^2 w + b(\frac{v}{w})^2$.

proof: Let $D=(d_{ij})$ be the given $(4,v;w)$-HDM over the group G with a hole H and $A=(a_{ij})$ a $CA(b;3,5,w)$ over H. Denote the set of cosets of H in G by $\mathcal{H}=\{g_1+H, g_2+H, \ldots, g_{v/w}+H\}$, where g_i is the representative of the i-th coset of H.
For each column $(d_{1j}, d_{2j}, d_{3j}, d_{4j})^T$ of the HDM, construct the following rows:

$$(d_{1j}+u, d_{2j}+u, d_{3j}+u+e, d_{4j}+u+e, e),$$

where $e, u \in G$.
For the r-th row $(a_{1r}, a_{2r}, a_{3r}, a_{4r}, a_{5r})$ of the $b \times 5$ array A $(1 \leq r \leq b)$, construct the following rows:

$$(a_{1r}+g_l, a_{2r}+g_l, a_{3r}+g_l+g_m, a_{4r}+g_l+g_m, a_{5r}+g_m),$$

where $1 \leq l, m \leq v/w$.

New upper bound on $\mathrm{CAN}(3, 5, v)$

1. For any odd positive integer v satisfying $gcd(v, 9) \neq 3$, $\mathrm{CAN}(3, 5, 2v) \leq 2v^2(4v + 1)$.

New upper bound on CAN$(3, 5, v)$

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v	Old bound	New bound
10	1219[1]	1050
12	1991[1]	1728
14	3107[1]	2824
18	6443[1]	5994
21	10100[1]	9261
22	12165[1]	10890
24	14927[1]	13824

[1] M. Chateauneuf and D. L. Kreher On the state of strength-three covering arrays, *J. Combin. Des.*, 10 (2002) 217-238.

3. CA$(3,6,v)$

DCA associated with an adder

Let $D=(d_{ij})$ be a DCA$(4,n;v)$ over an abelian group G. An n-tuple $s=(s_1,s_2,\ldots,s_n)$ over G is called an *adder* of the difference covering array D if each element of G appears in the multiset $\{s_1,s_2,\ldots,s_n\}$ at least once and the matrix

$$D^s=(d'_{ij}),\ \text{where}\ d'_{ij}=d_{ij}\ \text{for}\ i\in\{1,2\}\ \text{and}\ d'_{ij}=d_{ij}+s_j\ \text{otherwise}$$

is also a DCA$(4,n;v)$ over the group G.

3. CA$(3, 6, v)$

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Let $D = (d_{ij})$ be a DCA$(4, n; v)$ over an abelian group G. An n-tuple $s = (s_1, s_2, \ldots, s_n)$ over G is called an *adder* of the difference covering array D if each element of G appears in the multiset $\{s_1, s_2, \ldots, s_n\}$ at least once and the matrix

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is also a DCA$(4, n; v)$ over the group G.

1. A DCA$(4, 11; 10)$ over Z_{10}.

$$\left(\begin{array}{ccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 1 & 3 & 6 & 8 & 2 & 0 & 9 & 4 & 7 & 5 \\ 0 & 3 & 7 & 4 & 1 & 9 & 9 & 5 & 8 & 2 & 6 \\ \hline 0 & 6 & 1 & 2 & 4 & 3 & 7 & 2 & 5 & 9 & 8 \end{array}\right)$$

3. $CA(3, 6, v)$

DCA associated with an adder

Let $D = (d_{ij})$ be a $DCA(4, n; v)$ over an abelian group G. An n-tuple $s = (s_1, s_2, \ldots, s_n)$ over G is called an *adder* of the difference covering array D if each element of G appears in the multiset $\{s_1, s_2, \ldots, s_n\}$ at least once and the matrix

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1. A $DCA(4, 11; 10)$ over Z_{10}.

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 1 & 3 & 6 & 8 & 2 & 0 & 9 & 4 & 7 & 5 \\ 0 & 3 & 7 & 4 & 1 & 9 & 9 & 5 & 8 & 2 & 6 \\ \hline 0 & 6 & 1 & 2 & 4 & 3 & 7 & 2 & 5 & 9 & 8 \end{pmatrix}$$

2. For any prime power $q \geq 4$, there is a $(q, 4; 1)$-DM associated with an adder.

A CA$(nv^2; 3, 6, v)$ via DCA$(4, n; v)$

Construction 3

If there exists a DCA$(4, n; v)$ associated with an adder, then there exists a CA$(nv^2; 3, 6, v)$.

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Construction 3

If there exists a DCA(4, n; v) associated with an adder, then there exists a CA(nv^2; 3, 6, v).

*Proof.*Let $D = (d_{ij})$ and $s = (s_1, \ldots, s_n)$ be the given DCA(4, n; v) over the group G and its corresponding adder. A CA(nv^2; 3, 6, v) over G consists of the following rows:

$$(d_{1j} + u, d_{2j} + u, d_{3j} + u + e + s_j, d_{4j} + u + e + s_j, e, e + s_j),$$

where $(d_{1j}, d_{2j}, d_{3j}, d_{4j})^T$ is the jth column of the DCA, and $e, u \in G$.

A CA$(nv^2; 3, 6, v)$ via DCA$(4, n; v)$

Construction 3

If there exists a DCA$(4, n; v)$ associated with an adder, then there exists a CA$(nv^2; 3, 6, v)$.

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where $(d_{1j}, d_{2j}, d_{3j}, d_{4j})^T$ is the jth column of the DCA, and $e, u \in G$.

Corollary

If there is a $(v, 4; 1)$-DM associated with an adder, then there is an OA$(3, 6, v)$.

New OA$(3, 6, v)$

1. There is a $(v, 4; 1)$-DM associated with an adder for $v \in \{12, 15, 21, 24\}$.

New OA$(3, 6, v)$

1. There is a $(v, 4; 1)$-DM associated with an adder for $v \in \{12, 15, 21, 24\}$.
2. Let v be a positive integer which satisfies $\gcd(v, 4) \neq 2$ and $\gcd(v, 9) \neq 3$. Then there is an OA$(3, 6, v)$. (Ji and Yin)

New OA$(3, 6, v)$

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Question:

Construct a $(3p, 4; 1)$-DM associated with an adder for any prime $p \geq 11$?

New upper bound on $\mathrm{CAN}(3,6,v)$

1. For any integer $v \equiv 2 \ (mod\ 4)$ and $\gcd(v,9) \neq 3$, there exists a $\mathrm{DCA}(4,v+1;v)$ associated with adder.

New upper bound on CAN$(3, 6, v)$

1. For any integer $v \equiv 2 \ (mod\ 4)$ and $\gcd(v, 9) \neq 3$, there exists a DCA$(4, v+1; v)$ associated with adder.
2. For any integer $v \equiv 2 \ (mod\ 4)$ and $\gcd(v, 9) \neq 3$, CAN$(3, 6, v) \leq v^3 + v^2$.

New upper bound on CAN$(3,6,v)$

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v	Old bound	New upper bound
10		1100
12	2112[1]	1728
14	3289[1]	2940
18	6749[1]	6156
22		11132
24	15479[1]	13824

[1] M. Chateauneuf and D. L. Kreher On the state of strength-three covering arrays, *J. Combin. Des.*, 10 (2002) 217-238.

HDM associated with an adder

Let $D = (d_{ij})$ be a $(4, v; w)$-HDM over $(G; H)$. An $v-2$-tuple $s = (s_1, s_2, \ldots, s_{v-w})$ over G is called an *adder* of the holey difference matrix D if $G \setminus H = \{s_1, s_2, \ldots, s_{v-w}\}$ and the matrix

$$D^s = (d'_{ij}), \text{ where } d'_{ij} = d_{ij} \text{ for } i \in \{1, 2\} \text{ and } d'_{ij} = d_{ij} + s_j \text{ otherwise}$$

is also a $(4, v; w)$-HDM over $(G; H)$.

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Let $D = (d_{ij})$ be a $(4, v; w)$-HDM over $(G; H)$. An $v-2$-tuple $s = (s_1, s_2, \ldots, s_{v-w})$ over G is called an *adder* of the holey difference matrix D if $G \setminus H = \{s_1, s_2, \ldots, s_{v-w}\}$ and the matrix

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is also a $(4, v; w)$-HDM over $(G; H)$.

If there exists a $(4, v; w)$-HDM associated with an adder and a CA$(b; 3, 6, w)$, then CAN$(3, 6, v) \leq v^3 - v^2 w + b(\frac{v}{w})^2$.

HDM associated with an adder

Let $D = (d_{ij})$ be a $(4, v; w)$-HDM over $(G; H)$. An $v - 2$-tuple $s = (s_1, s_2, \ldots, s_{v-w})$ over G is called an *adder* of the holey difference matrix D if $G \setminus H = \{s_1, s_2, \ldots, s_{v-w}\}$ and the matrix

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If there exists a $(4, v; w)$-HDM associated with an adder and a CA$(b; 3, 6, w)$, then CAN$(3, 6, v) \leq v^3 - v^2 w + b(\frac{v}{w})^2$.

For any integer $v \equiv 2 \ (mod\ 4)$ and $\gcd(v, 9) \neq 3$, there exists a $(4, v; 2)$-HDM associated with an adder.

4. CA$(4, 6, v)$

Definition: Special $(4, v; w)$-HDM

Let $D = (d_{ij})$ be a $(4, v; w)$-HDM over $(G; H)$. If the difference list
$\Delta_P = \{a_{1j} + a_{4j} - a_{2j} - a_{3j} : 1 \leq j \leq v - w\} = G \setminus H$, then this HDM is denoted by $(4, v; w)$-HDM*.

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There is a $(4, 2p; 2)$-HDM* for any prime $p \equiv 1$ $(mod\ 4)$ with $p \geq 13$.

A CA$(4, 6, v)$ from special $(4, v; w)$-HDM

Construction 4

If there exists a $(4, v; w)$-HDM* and a CA$(b; 4, 6, w)$, then CAN$(4, 6, v) \leq v^4 - wv^3 + b(\frac{v}{w})^3$.

A CA$(4,6,v)$ from special $(4,v;w)$-HDM

Construction 4

If there exists a $(4,v;w)$-HDM* and a CA$(b;4,6,w)$, then CAN$(4,6,v) \leq v^4 - wv^3 + b(\frac{v}{w})^3$.

Proof: Let $D=(d_{ij})$ be the given $(4,v;w)$-HDM* over $(G;H)$ and $A=(a_{ij})$ a CA$(b;4,6,w)$ over H. Denote the set of cosets of H in G by $\mathcal{H}=\{g_1+H, g_2+H, \ldots, g_{v/w}+H\}$, where g_i is the representative of the i-th coset of H.

For each column $(d_{1j},d_{2j},d_{3j},d_{4j})^T$ of the HDM, construct the following rows:

$$(d_{1j}+u, d_{2j}+u+w, d_{3j}+u+e, d_{4j}+u+e+w, e, w),$$

where $e,u,w \in G$.

For each row $(a_{1r},a_{2r},a_{3r},a_{4r},a_{5r},a_{6r})$ of the $b \times 6$ array A, construct the following rows:

$$(a_{1r}+g_l, a_{2r}+g_l+g_n, a_{3r}+g_l+g_m, a_{4r}+g_l+g_m+g_n, a_{5r}+g_m, a_{6r}+g_n),$$

where $1 \leq l,m,n \leq v/w$.

New bound on CA$(4, 6, v)$

For any prime $p \equiv 1 \ (mod\ 4)$ with $p \geq 13$, CAN$(4, 6, 2p) \leq 16p^4 + 5p^3$.

Definition: Special DCA$(4, n; v)$

Let $D = (d_{ij})$ be a DCA$(4, n; v)$ over G. If the multiset $\Delta_P = \{a_{1j} + a_{4j} - a_{2j} - a_{3j} : 1 \leq j \leq n\}$ contain each element of G at least once, then this DCA is denoted by DCA$^*(4, n; v)$.

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There is a DCA$^*(4, 2p + 2; 2p)$ for any prime $p \equiv 1 \ (mod\ 4)$ with $p \geq 13$.

If there exists a DCA$^*(4, n; v)$, then CAN$(4, 6, v) \leq nv^3$.

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If there exists a DCA$^*(4, n; v)$, then CAN$(4, 6, v) \le nv^3$.

Question: Construct a $(3p, 4, 1)$-DM* for prime $p \ge 7$.

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Thank you!