Constructive characterizations of (γ_p, γ)- and (γ_p, γ_{pr})-trees

Chen Lei

Shanghai Key Laboratory of Trustworthy Computing
East China Normal University

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Domination

A set $S \subseteq V$ is a *dominating set* in a graph $G = (V, E)$ if every vertex in $V - S$ has at least one neighbor in S, that is $N[S] = V$.

The *domination number* of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G.

A dominating set of G with minimum cardinality is called a *$\gamma(G)$-set*.

Power domination

A subset $S \subseteq V$ is a *power dominating set* of G if all vertices of V can be observed recursively by the following rules:
(i) all vertices in $N[S]$ are observed initially,
(ii) if an observed vertex u has all neighbors observed except one neighbor v, then v is observed (by u).

The *power domination number*, denoted by $\gamma_p(G)$, is the minimum cardinality of a power dominating set of G.

A power dominating set of G with minimum cardinality is called a *$\gamma_p(G)$-set*.

Paired-domination

A set $S \subseteq V$ is a *paired-dominating set* of G if S is a dominating set of G and the induced subgraph $G[S]$ has a perfect matching.

The *paired-domination number*, denoted by $\gamma_{pr}(G)$, is defined to be the minimum cardinality of a paired-dominating set S in G.

A paired-dominating set of G with minimum cardinality is called a *$\gamma_{pr}(G)$-set*.

An area of research in domination of graphs that has received considerable attention is the study of classes of graphs with equal domination parameters. For any two graph theoretic parameters λ and μ, G is called a *(λ, μ)-graph* if $\lambda(G) = \mu(G)$.

In this lecture, constructive characterizations of (γ_p, γ)-trees and (γ_p, γ_{pr})-trees will be given.

Operations of (γ_p, γ)-trees

Type-1 **operation:** Attach a P_1 to a strong support vertex v of T.

Type-2 **operation:** Attach a P_3 to T by adding a path $wy'y$ or wy, where w is not a strong leaf in T, y is a support vertex of P_3 and y' is neither in P_3 nor in T.

A vertex of T is said to be a *strong support vertex* if it is adjacent to at least two leaves.
Let v be a leaf of T and uv be an edge of T. We called v to be a *strong leaf* if there are at most two leaves in $N(u)$.

Some lemmas I

Lemma 1 For any tree T with at least three vertices,
(i) If $\gamma_p(T) = \gamma(T)$, then every support vertex is a strong support vertex.
(ii) Let $SS(T)$ be the set of all strong support vertices of T. Then, $\gamma_p(T) = \gamma(T)$ if and only if $SS(T)$ is a domination set of T.

Lemma 2 $\mathcal{F} \subseteq \mathcal{T}$.

Lemma 3 Let $T \in \mathcal{T}$ be a tree with at least three vertices. Then, $T \in \mathcal{F}$.

Where $\mathcal{T} = \{T \mid \gamma_p(T) = \gamma(T)\}$ and $\mathcal{F} = \{T \mid T$ is obtained from P_3 by a finite sequence of operations of type-1 or type-2 $\}$.

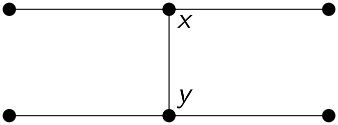
Main results I

Theorem 1 $\mathcal{F} \cup \{P_1, P_2\} = \mathcal{T}$.

Operations of (γ_p, γ_{pr})-trees

Type-1 **operation:** Attach a P_1 to a strong support vertex v of T.

Type-3 **operation:** Attach the tree T^* in Figure 1 to T by adding a path $wy'y$ or wy, where w is not a strong leaf of T and y' is neither in T^* nor in T.



T^*

Some lemmas II

Lemma 4 For any tree T, $\gamma_p(T) = \gamma_{pr}(T)$ if and only if $SS(T)$ is a domination set of T and the subgraph induced by $SS(T)$ has a perfect matching.

Lemma 5 $\mathcal{F}_p \subseteq \mathcal{T}_p$.

Lemma 6 $\mathcal{T}_p \subseteq \mathcal{F}_p$.

Where $\mathcal{T}_p = \{T \mid \gamma_p(T) = \gamma_{pr}(T)\}$ and $\mathcal{F}_p = \{T \mid T$ is obtained from T^* by a finite sequence of operations of type-1 or type-3.$\}$.

Main results II

Theorem 2 $\mathcal{T}_p = \mathcal{F}_p$.

Thank you!