Degree Conditions of Fractional ID-k-factor-critical Graphs

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Introduction

Some Results About Fractional Factor

The Main Results About Fractional ID-k-critical Graphs

1. Introduction

- The graphs considered in this paper will be finite and undirected simple graphs. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The minimum degree of G is denoted by $\delta(G)$. For any vertex x of G, the degree of x is denoted by $d_G(x)$.
- We use $G[S]$ and $G - S$ to denote the subgraph of G induced by S and $V(G) - S$, respectively, for $S \subseteq V(G)$.

- A subset I of $V(G)$ is said to be independent if no two distinct vertices in I are adjacent. A $matching$ in a graph is a set of edges no two of which are adjacent. A matching is $perfect$ if it covers all vertices of the graph.
- A graph G is factor-criticalif $G-v$ has a perfect matching for every vertex $v \in V(G)$. We say that G is independent-set-deletable factor-critical (shortly, ID-factor-critical) if for every independent set I of G which has the same parity with $V(G)$, $G - I$ has a perfect matching.

- Let $f : E(G) \longrightarrow [0, 1]$ be a real-valued function from the edge set $E(G)$ to the real number interval $[0, 1]$. For any $e \in E(G)$, $f(e)$ is referred to the *weight* of the edge e. Define $E_f = \{e \in E(G) : f(e) > 0\}$. If $\sum f(e) = k$ is satisfied for each vertex $v \in V(G)$ where the sum is taken over all edges incident to v, then $G[E_f]$, or G_f for short, is called a factional k-factor of G with indictor function f.

- A fractional 1-factor is also called a factional perfect matching. We say that G is fractional ID k-factor-critical if for every independent set I of $V(G)$, $G-I$ has a fractional k-factor. When $k=1$, we say that G is fractional ID-factor-critical if for every independent set I of $V(G)$, $G-I$ has a fractional prefect matching. In this paper, we study the degree conditions of fractional ID-factor-critical graphs.

2. Some Results About Fractional Factor

- Liu and Zhang gave necessary and sufficient conditions for a graph to have fractional k-factor.

Lemma 2.1 Let G be a graph. Then G has a fractional k-factor if and only if for every subset S of $V(G)$, $\Phi_G(S, T, k) \geq 0$, where $\Phi_G(S, T, k) = k|S| - k|T| + d_{G-S}(T)$, $T = \{x : x \in V(G) - S, d_{G-S}(x) \leq k\}$.

- **Lemma 2.2** Let G be a graph. Then G has a fractional k-factor if and only if for every subset S of $V(G)$, $k|S| - \sum_{0\le i\le k-1}(k-i)p_i(G\backslash S) \geq 0$, where $p_i(G\backslash S) = |\{x : x \in V(G)\backslash S, d_{G\backslash S}(x) = i\}|$.

3. The Main Results About Fractional ID-k-critical Graphs

- We have the following results about ID-factor-critical graphs.

Lemma 3.1. Let G be a graph with $n \geq 3$ vertices. Then if $\delta(G) \geq \frac{n}{2}$. Then G is hamiltonian.

- **Lemma 3.2.** Let G be a graph of order n and $\delta(G) \geq \frac{2n}{3}$, then G is fractional ID k-factor-critical when $k = 1, 2$.

- Now we give the degree conditions of fractional ID-k-factor-critical graphs when $k \geq 3$.

Theorem 3.3. Let k be a positive integer and G be a graph of order n with $n \geq 6k - 8$. If $\delta(G) \geq \frac{2n}{3}$, then G is fractional ID-k-factor-critical.

- In this section we show that the conditions in Theorem 3.3 are best possible.

Theorem 3.4. The bound of n is best possible in some sence.

- The bound of $\delta(G)$ is sharp indeed. To see this, we could construct a graph G with minimum degree $\lceil \frac{2n}{3} \rceil - 1$ which is not fractional ID-k-factor-critical.

Theorem 3.5. $\lceil \frac{2n}{3} \rceil$ is the minimum integer δ such that every graph with $\delta(G)$ at least δ and $n \geq 6k - 8$ is fractional ID k-factor-critical.

References

[1] J.A. Bondy and U.S.R. Murty. Graph Theory with Applications. Macmillan, London, 1976.

[2] Edward R. Scheinermman and Ullman Daniel H. Fractional Graph Theory. New York: John and Wiley and Sons, Inc., 1997.

[3] Lovasz, L. and M. D. Plummer. Matching Theory. Elsevier Science Publishers, B. V. North Holland, 1985.

[4] Liu Yan and Ma Yinghong. Degree Conditions of ID-factor-critical Graphs. Southeast Asian Bulletin of Mathematics, 2003, 27: 641–648.

[5] Liang Caixia and Liu Yan. The Degree Sum Condition of ID-factor-critical Graphs. Chinese Journal of Engineering Mathematics, 2006, 23: 169–174.

[6] Yuan J J. The independent-set-deletable factor-critical graphs[R]. submitted.

[7] Yu Jiguo, Liu Guizhen, Ma Meijie and Cao Baoxiang. A Degree Condition for Graphs to Have Factional Factors. Advances in Mathematics. 2006, 35: 621–628.

[8] Liu Guizhen and Zhang Lanju. Fractonal (g,f)-factors of graphs. Acta Mathematica Scientia, 2001, 21(B)1: 541–545.

[9] Zhang Lanju and Liu Guizhen. Fractonal k-factors of graphs. J. Sys. Sci. and Math. Scis., 2001, 21(1): 88-92.

Thank you!