# COMPLEX GEOMETRY AND BEYOND

in honor of Ngaiming Mok

East China Normal University, Shanghai

2025.05.31 - 2025.06.04





### **Speakers**

Fudan U., China **Meng Chen** C.N.R.S., France Ya Deng

U. Illinois Chicago, United States Lawrence Ein

U. Grenoble Alpes, France Philippe Eyssidieux

C.A.S., China Baohua Fu

**Ziyang Gao** Samuel Grushevsky Stony Brook U., United States

U.C.L.A., United States

I.B.S., Korea Jun Muk Hwang Fudan U., China Qingchun Ji I.B.S., Korea Sung Yeon Kim

Humboldt U. Berlin, Germany Bruno Klingler

Shandong U., China Qifeng Li U. Tokyo, Japan Keiji Oguiso

Harvard U., United States Yum-Tong Siu

E.C.N.U., China Sheng-li Tan I.H.É.S., France **Emmanuel Ullmo** Peking U., China Junyi Xie

Purdue U., United States Sai-Kee Yeung\*

Wuhan U., China Wanke Yin Peking U., China Xinyi Yuan Nankai U., China Weiping Zhang C.A.S., China Xiangyu Zhou Wuhan U., China Kang Zuo

### **Schedule**

Venue: Rm.102, Mathematics Building, East China Normal University

	May 31	June 1	June 2	June 3	June 4
9:00-9:50	Registration (9:00-9:20)  Opening ceremony and group photo (9:20-9:50)	Zhang	Klingler	Zhou	Ein
9:50-10:20	Tea Time				
10:20-11:10	Siu	Oguiso	Yuan	Xie	Chen
11:10-11:20	Break				
11:20-12:10	Zuo	Fu	Gao	Deng	Tan
12:10-2:30	Lunch (hotel)				
2:30-3:20	Hwang	Eyssidieux	Ullmo	Ji	
3:20-3:50	Tea Time				
3:50-4:40	Kim	Grushevsky	Mok (colloquium)	Yin	
4:40-4:50	Bre	ak	Diamon		
4:50-5:40	Li	Yeung*	Dinner (Platinum Hanjue Hotel	Dinner (campus	
5:40-7:00	Dinner (campus canteen)		白金汉爵)	canteen)	

### **Arrangements**

# Shuttles will be arranged between the hotel and Mathematics building as follows:

From hotel to Math. building:

Morning: May 31 – Jun 4 (8:30 am) Afternoon: May 31 – Jun 3 (2:00 pm)

From Math. building to hotel:

May 31, Jun 1, Jun 3 (7:00 pm)

#### **Lunch and dinner**

Lunch will be at the hotel and dinner (except on Jun 2) will be in a campus canteen next to Mathematics building.

For lunch, we will go together by shuttles after the morning session.

For dinner in the campus canteen (served between 4:30pm to 7:00pm), please pay with the vouchers we have provided for you.

On Jun 2, we will have dinner at Platinum Hanjue Hotel (白金汉爵), whose address is:

No. 1577 Humin Road, Minhang District (闵行区沪闵路 1577 号)

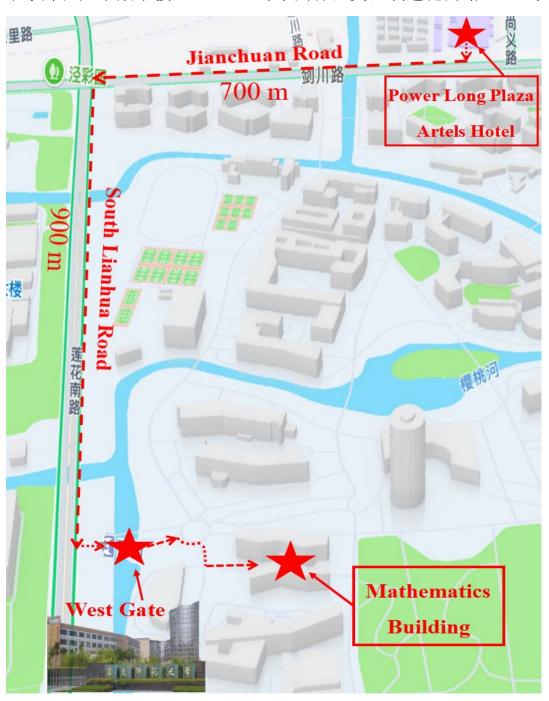
#### **Local contacts:**

Sui-Chung Ng 吴瑞聪+86-13621817050Yun Gao高云+86-13761788214Zhangchi Chen 陈张弛+86-18001356821

### **Directions**

**Hotel:** Artels, No.39-1, Shangyi Road, Minhang District, Shanghai 宝龙艺悦酒店 上海市闵行区吴泾镇尚义路 39 弄 1 号

**Conference Venue:** Rm.102, Mathematics Building, East China Normal University, 5005 South Lianhua Road, Minhang District, Shanghai 华东师范大学数学楼 102 室 上海市闵行区吴泾镇莲花南路 5005 号



#### **Abstracts**

#### The Noether inequality of algebraic 3-folds and related moduli spaces

Meng Chen

Fudan University

**Abstract:** In this lecture, I will provide an outline of the proof for the following theorem: the inequality  $K^3 \ge \frac{4}{3}p_g - \frac{10}{3}$  holds for any complex 3-fold of general type. Based on this inequality, I will explain an effective method to study moduli spaces of canonical 3-folds with boundary volumes.

#### **Deformation of Algebraic Varieties with Big Fundamental Groups**

Ya Deng

French National Center for Scientific Research

Abstract: In 2002, De Oliveira, Katzarkov, and Ramachandran conjectured that a small smooth deformation of a complex projective variety X with a big fundamental group should still have a big  $\pi_1$ , in analogy with Siu's invariance of plurigenera. Over the past two decades, this conjecture had only been proved for surfaces and, under suitable conditions, for threefolds by Benoit Claudon. In recent joint work with Chikako Mese and Botong Wang, we proved this conjecture in the case where  $\pi_1(X)$  is linear, i.e., admits an almost faithful linear representation. In this talk, I will outline the main ideas of proof and discuss related results, including Campana's broader conjecture on the deformation invariance of the  $\Gamma$ -dimension, and its application to the deformation openness of pseudo Brody hyperbolicity.

#### Syzygies and singularities of secant varieties of algebraic curves

Lawrence Ein

University of Illinois Chicago

**Abstract:** We'll discuss joint work with Wenbo Niu and Jinhyung Park on the syzgygies of singulariteis of secant varieties of smooth projective curves..

#### Algebrogeometric subgroups of the mapping class group

Philippe Eyssidieux

Université Grenoble Alpes

**Abstract:** Joint work with Louis Funar. We introduce a method to provide constraints for algebrogeometric subgroups of mapping class groups, i.e.: images of fundamental groups of curves under complex algebraic maps to the moduli space of smooth curves, which consists in studying the Shafarevich morphisms attached to the orbifold compactifications constructed in our previous work (arXiv:2112.06726). This implies for instance that these algebrogeometric subgroups have infinite image under quantum representations except in a few well-understood cases.

#### Symplectic singularities arising from cotangent bundles

Baohua Fu

Chinese Academy of Sciences

**Abstract:** I'll report joint works with Jie Liu (AMSS) constructing symplectic singularities from the affinization of cotangent bundles of smooth varieties.

## Generic positivity of the Beilinson-Bloch height of Gross-Schoen and Ceresa cycles

Ziyang Gao

University of California Los Angeles

**Abstract:** Given an algebraic curve defined over a number field, one can define the Néron-Tate height on the Jacobian and prove its positivity. This height pairing and its positivity play important roles in the proof of the Mordell-Weil theorem, in Vojta's proof of the Mordell conjecture, and in the formulation of the BSD conjecture. The Jacobian can be seen, via the Abel-Jacobi map, as the moduli space of 0-cycles of degree 0 on the algebraic curve.

The analogue for higher cycles was studied by Weil, Griffiths, Beilinson, and Bloch. In particular in the 1980s, Beilinson and Bloch independently proposed a conditional definition of heights for arbitrary homologically trivial cycle. The positivity of their heights, as conjectured by Beilinson and Bloch, is widely open.

In this talk, I will report a recent joint work with Shouwu Zhang about a generic positivity for the Gross-Schoen and Ceresa cycles of curves of genus at least 3. These are the simplest situation where the Beilinson-Bloch heights are unconditionally defined.

#### Maximal compact subvarieties of moduli spaces

Samuel Grushevsky

Stony Brook University

**Abstract:** We present results on the maximal dimension of compact subvarieties of the moduli space of abelian varieties and of moduli of complex curves of compact type. Equivalently, this is the maximal dimension of a compact complex parameter space for a maximally varying family of abelian varieties/curves, etc. Based on joint work with Mondello, Salvati Manni, Tsimerman.

#### Minimal rational curves on equivariant group compactifications

Jun Muk Hwang

Institute for Basic Science

**Abstract:** Let X be a nonsingular equivariant compactification of a simple algebraic group G. We show that minimal rational curves on X are orbit-closures of 1-parameter subgroups of G and the set of minimal rational curves through a general point is the closure of an adjoint orbit. This generalizes a result of Brion and Fu's on wonderful group compactifications to arbitrary equivariant group compactifications. This is a joint work with Qifeng Li.

#### A notion of convexity for Levi-flat structures

Qingchun Ji

Fudan University

**Abstract:** The Levi-flat structure originates from the seminal work of L. Nirenberg generalizing the Newlander-Nirenberg theorem on complex structures. Since then, it has become a central topic in the theory of involutive structures. In this work, we introduce a notion of convexity for Levi-flat structures, inspired by Morse theory and Grauert-type convexity from Several Complex Variables. Applications to the global and local solvability of the Treves complex will be presented. We will also talk about the extension problem for the canonical bundle associated with a Levi-flat structure.

### Proper holomorphic maps between bounded symmetric domains with small rank differences

Sung Yeon Kim

Institute for Basic Science

**Abstract:** In this talk, we study the rigidity of proper holomorphic maps  $f:\Omega\to\Omega'$  between irreducible bounded symmetric domains  $\Omega$  and  $\Omega'$  with small rank differences:  $2 \le \operatorname{rank}(\Omega') < 2\operatorname{rank}(\Omega) - 1$ . More precisely, if either  $\Omega$  and  $\Omega'$  have the same type or  $\Omega$  is of type~III and  $\Omega'$  is of type~I, then up to automorphisms, f is of the form  $f = \iota \circ F$ , where  $F = F_1 \times F_2 : \Omega \to \Omega'_1 \times \Omega'_2$ . Here  $\Omega'_1$ ,  $\Omega'_2$  are bounded symmetric domains, the map  $F_1 : \Omega \to \Omega'_1$  is a standard embedding,  $F_2 : \Omega \to \Omega'_2$ , and  $\iota : \Omega'_1 \times \Omega'_2 \to \Omega'$  is a totally geodesic holomorphic isometric embedding. As a consequence,  $f : \Omega \to \Omega'$  is a holomorphic totally geodesic isometric embedding with respect to Kobayashi metrics. Moreover we show that, under the rank condition above, there exists no proper holomorphic map  $f : \Omega \to \Omega'$  if  $\Omega$  is of type~I and  $\Omega'$  is of type~III, or  $\Omega$  is of type~II and  $\Omega'$  is either of type~I or III. This is a joint work with N. Mok and A. Seo.

#### Special loci for algebraic varieties

Bruno Klingler

Humboldt University of Berlin

**Abstract:** I will discuss various special loci for algebraic varieties related to their fundamental groups, their properties, and their (conjectural) relations.

#### The geometric structures associated with VMRT-structures

Qifeng Li

Shandong University

**Abstract:** The local structures of VMRT's (the varieties of minimal rational tangents) carry much information on global geometry of manifolds. A typical example is the Cartan-Fubini type extension theorem due to Hwang and Mok, which indicates that Fano manifolds of Picard number one can be determined by their local VMRT-structures. We are interested in isotrivial VMRT-structures, the simplest local VMRT-structures. In this talk, we will discuss on the geometric structures associated with the isotrivial VMRT-structures as well as the applications in algebraic geometry. This talk is based on joint works with Jun-Muk Hwang.

## Starting with the Gauss-Bonnet formula: rigidity phenomena on bounded symmetric domains

Ngaiming Mok

The University of Hong Kong

**Abstract:** Let E be a compact Riemann surface of genus 1, and Z be a compact Riemann surface of genus  $\geq 2$ . Then, every holomorphic map  $f:E\to Z$  is constant, as can be proven by contradiction by pulling back a nontrivial holomorphic differential on Z which necessarily vanishes at some point. A metric version of the proof using the Gauss-Bonnet formula is more flexible, and a variation of the proof based on a Chern integral gives a Hermitian metric rigidity theorem, first established by the author in 1987 in the case of compact quotients  $X_{\Gamma} \coloneqq \Omega / \Gamma$  of irreducible bounded symmetric domains  $\Omega$  of rank  $\geq 2$  and then extended in the finite-volume case by To in 1989, which gives rigidity results on holomorphic maps from  $X_{\Gamma}$  to Kähler manifolds of nonpositive holomorphic bisectional curvature, and geometric superrigidity results in the special cases of  $\Gamma \setminus G/K$  for G/K of Hermitian type and of rank  $\geq 2$  and for cocompact lattices  $\Gamma \subset G$  via the use of harmonic maps and the  $\partial \overline{\partial}$ -Bochner-Kodaira formula of Siu's in 1980.

The Hermitian metric rigidity theorem was the starting point of the author's investigation on rigidity phenomena mostly on bounded symmetric domains  $\Omega$ irreducible of rank  $\geq 2$ , but also, in the presence of irreducible lattices  $\Gamma \subset G := \operatorname{Aut}_0(\Omega)$ , on reducible  $\Omega$ , and, for certain problems also on the rank-1 cases of n-dimensional complex unit ball  $\mathbb{B}^n$ . The proof of Hermitian metric rigidity serves both (I) as a prototype of metric rigidity theorems and (II) as a source for proving rigidity results or making conjectures on rigidity phenomena for holomorphic maps. For type-I results the author will explain (1) the finiteness theorem on Mordell-Weil groups of universal polarized Abelian varieties over function fields of Shimura varieties, established by Mok (1991) and by Mok-To (1993), (2) a Finsler metric rigidity theorem of the author's (2004) for quotients  $X_{\Gamma} := \Omega / \Gamma$  of bounded symmetric domains  $\Omega$  of rank  $\geq 2$  by irreducible lattices and a recent application by He-Liu-Mok (2024) proving the triviality of the spectral case when  $X_{\Gamma}$  is compact, (3) a rigidity result of Clozel-Ullmo (2003) characterizing commutants of certain Hecke correspondences on irreducible bounded symmetric domains  $\Omega$  of rank  $\geq 2$  via a reduction to a characterization of holomorphic isometries which follows from the proof of Hermitian metric rigidity. For type-II results the author will focus on irreducible bounded symmetric domains  $\Omega$  of rank  $\geq 2$  and explain (4) the rigidity results of Mok-Tsai (1992) on the characterization of realizations of  $\Omega$  as convex domains in Euclidean spaces, (5) its ramification to a rigidity result of Tsai's (1994) on proper holomorphic maps in the equal rank case, (6) a semi-rigidity theorem of Kim-Mok-Seo (2025) on proper holomorphic maps between irreducible bounded symmetric domains of rank  $\geq 2$  in the non-equi-rank case, and (7) a theorem of Mok-Wong (2023) characterizing Γ-equivariant holomorphic maps into arbitrary

bounded domains inducing isomorphisms on fundamental groups.

Through Hermitian metric rigidity the author wishes to highlight the fact that complex differential geometry links up with many research areas of mathematics, as illustrated for instance by the aforementioned results (6) of Kim-Mok-Seo on proper holomorphic maps in which techniques of several complex variables cross-fertilize with those in CR geometry and the geometric theory of varieties of minimal rational tangents (VMRT), and (7) of Mok-Wong in which harmonic analysis meets ergodic theory and Kähler geometry.

### On the Kawaguchi--Silverman Conjecture for birational automorphisms of irregular varieties

Keiji Oguiso

University of Tokyo

**Abstract:** We would like to discuss the main open parts of the Kawaguchi--Silverman Conjecture, asserting that for a birational self-map f of a smooth projective variet X defined over  $\overline{\mathbb{Q}}$ , the arithmetic degree  $\alpha_f(x)$  exists and coincides with the first dynamical degree  $\delta_f$  for any closed point x of X with a Zariski dense orbit. Among other results, we show that this holds when X has Kodaira dimension zero and irregularity  $q(X) \ge \dim(X) - 1$  or X is an irregular threefold (modulo one possible exception). We discuss then the existence of Zariski dense orbits, with explicit examples. This talk is based on a joint work with Professors Jungkai Chen and Huesh-Yung Lin.

#### Global Nondeformability, Rigidity and Hyperbolicity

Yum-Tong Siu

Harvard University

**Abstract:** Will discuss topics in global nondeformability, rigidity and hyperbolicity in the context of topological, deformational, metric, curvature, number-theoretical conditions. Will start with the background and motivation for such topics and look at the problems, techniques and recent results.

#### **TBA**

Sheng-li Tan

East China Normal University

**Ball quotients: an overview** 

Emmanuel Ullmo

Institut des Hautes Études Scientifiques

**Abstract:** We will survey some results concerning properties of Ball quotients. Some of them are due to Ngaiming Mok and others were obtained in recent works in collaboration with Gregorio Baldi. Ball quotients are quotients of the unit ball in a finite dimensional complex vector space by a lattice. They are the only examples of hermitian locally symmetric spaces of rank 1 and Margulis' result on arithmeticity and super-rigidity can't be applied making their study quite challenging.

#### Genus of dynatomic curves tends to infinite

Junyi Xie

Peking University

**Abstract:** Given a non-isotrivial one-parameter family of rational functions, we prove that the genus of dynatomic curves tends to infinity. As a consequence, we prove a geometric version of the Uniform Boundedness Conjecture proposed Morton-Silverman for one-parameter families of rational functions. We also proved the higher dimensional version under mild conditions. Our proof uses arithmetic equidistribution, woven currents, and bifurcation theory.

## On mappings between moduli spaces of curves and locally Hermitian symmetric spaces

Sai-Kee Yeung\*

Purdue University

**Abstract:** The goal of our talk is to explain some results about rigidity and classification of holomorphic mappings between moduli spaces of curves and locally Hermitian symmetric spaces.

#### Bounding smooth Levi-flat hypersurfaces in a Stein manifold

Wanke Yin

Wuhan University

**Abstract:** In this talk, we will focus on the problem of constructing a smooth Levi-flat hypersurface locally or globally attached to a real codimension two submanifold in  $\mathbb{C}^{n+1}$ , or more generally in a Stein manifold, with elliptic CR singularities. This is a question left open from the work of Dolbeault-Tomassini-Zaitsev, or a generalized version of a problem already asked by Bishop in 1965. Our study reveals an intricate interaction of Several Complex Variables with Symplectic Geometry and Foliation Theory. This is based on the recent joint work with H. Fang, X. Huang and Z. Zhou.

#### On the geometric Bogomolov conjecture

Xinyi Yuan

Peking University

**Abstract:** The Bogomolov conjecture over number fields was proved by Ullmo and Zhang, the geometric Bogomolov conjecture was proved by the works of Gubler, Yamaki, Contat--Gao--Habegger--Xie, and Xie--Yuan, and a generalization from abelian varieties to semi-abelian varieties is recently proved by Luo--Yu. In this talk, I will talk about these cases, and try to explain the algebraic geometry behind Xie--Yuan's case.

#### Positive scalar curvature on manifolds and foliations

Weiping Zhang

Nankai University

**Abstract:** The existence of Riemannian metrics of positive scalar curvature on a manifold is an important problem in differential geometry. For example it is closely related to the positive mass theorem in general relativity. We will discuss some recent progress in this subject.

# $L^2$ estimates for $\overline{\partial}$ equations and bundles with singular positive curvature

Xiangyu Zhou

Chinese Academy of Sciences

Abstract: In this talk, we first recall some basic properties of multiplier ideal sheaves associated to pseudoeffective line bundles (or psh functions), e.g., a solution of Demailly's strong openness conjecture (Guan-Zhou), then present a characterization of Nakano positivity via solving  $\overline{\partial}$  equations with  $L^2$  estimates (Deng-Ning-Wang-Zhou), which is a converse proposition of Hörmander-Demailly's  $L^2$  existence theorems. This gives a connection between differential geometry and differential equation via several complex variables. As an application of the characterization, we give an affirmative answer to Lempert's problem (Liu-Yang-Zhou), which asks whether the limit metric of an increasing sequence of hermitian metrics with Nakano semi positive curvature on holomorphic vector bundles is still Nakano semi-positive. As another application, one may define singular metric of positive curvature in the sense of Nakano on holomorphic vector bundles. Finally, we present recent results on the strong openness of the multiplier submodule sheaves (vector bundle version of multiplier ideal sheaves) by Liu-Xiao-Yang-Zhou and Le Poiter type isomorphism theorem between cohomology of the vector bundles twisted with the multiplier submodule sheaves and cohomology of the associated line bundles twisted with the multiplier ideal sheaves (Liu-Liu-Yang-Zhou).

#### Loci of non-rigid families of varieties in the corresponding moduli space

Kang Zuo

Wuhan University

**Abstract:** Inspired by the Bombieri-Lang conjecture, we propose a program studying the loci of non-constant and non-rigid maps into moduli spaces of varieties. We conjecture that if a "general" moduli space is not birational to any Shimura variety of rank > 1, then the loci of non-constant and non-rigid maps is contained in a proper subvariety of the moduli space. Under the assumption of a locally injective Torelli map, we find some evidence of this conjecture, which are consequences of the recent work by Baldi-Klingler-Ullmo on the distribution of Hodge loci. Conjectures of Ax-Schanuel type are expected to play an important role in the program. This is a joint project with Ke Chen, Tianzhi Hu and Ruiran Sun.