偏微分方程青年学者研讨会

华东师范大学数学科学学院

中国·上海

2022 年 11 月 5 日 — 11 月 6 日

偏微分方程青年学者研讨会
会议通知

　　我们将于 2022 年 11 月 5 日至 11 月 6 日举办"偏微分方程青年学者研讨会"。本次会议拟邀请偏微分方程领域优秀青年学者进行学术交流，探讨学术前沿问题，促进学术交流与合作。学术会议的主题涉及变分方法、临界点理论、非线性偏微分方程等相关领域。

会议地点： 腾讯会议：833 1462 6959　　密码: 221105

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会议日程

主持人	时间	报告信息
周风	08:50—09:00	开幕式
徐露	09:00—09:40	Regularity theory for the Dirichlet problem of porous media equations and fast diffusion equations 熊金钢 (北京师范大学)
魏龙	09:45—10:25	Periodic solutions of fractional Allen-Cahn equation 杜卓然 (湖南大学)
王征平	10:30—11:10	On isolated singularities for fractional Emden equations 陈虎元 (江西师范大学)
王小六	11:15—11:55	Rigidity and gap theorem for Liouville's equation 沈伟明 (首都师范大学)
		午　　休
刘勇	14:00—14:40	Co-axial metrics on the sphere and algebraic numbers 陈志杰 (清华大学)
杨敏波	14:45—15:25	Some progress on nonlocal differential inequalities on manifolds 孙玉华 (南开大学)
高琦	15:30—16:10	Uniqueness of single peak solutions for coupled nonlinear Gross-Pitaevskii equations with potentials 曾小雨 (武汉理工大学)
田如顺	16:15—16:55	Saddle solutions for the fractional Choquard equation 夏健康 (西北工业大学)

主持人	时间	报告信息
杨军	09:00—09:40	The blow-up analysis on $B_2^{(1)}$ affine Toda system: local mass and affine Weyl group 杨文 (中科院精密测量科学与技术创新研究院)
王俊	09:45—10:25	Uniqueness and existence results of mean field equations 胡烨耀 (中南大学)
王春花	10:30—11:10	Sign-changing solutions for the nonlinear Schrödinger-Poisson system 帅伟 (华中师范大学)
唐岚	11:15—11:55	The spectral gap to torsion problem for some non-convex domains 罗鹏 (华中师范大学)
午 休		
敖微微	14:00—14:40	Infinitely many nonradial positive solutions for multi-species nonlinear Schrödinger systems in $\mathbb{R}^N$ 吴元泽 (中国矿业大学)
陈文晶	14:45—15:25	Smooth interior approximation of sets and traces of weakly regular vector fields on rough domains 李沁峰 (湖南大学)
张贻民	15:30—16:10	On two phase transition problems and the exact solutions 罗森平 (江西师范大学)
郭千桥	16:15—16:55	Group invariant solutions for the planar Schrödinger-Poisson equation 周刚龙 (华东师范大学)

报告摘要 (报告人字母序)

On isolated singularities for fractional Emden equations

陈虎元　江西师范大学

In this talk, we show the isolated singularities of positive solutions to the fractional Emden Equations. Our analysis of isolated singularities is based on an integral upper bounds and he study of the Poisson problem with the fractional Hardy operators. This is joint work with prof. Laurent Veron and Prof. Feng Zhou.

Co-axial metrics on the sphere and algebraic numbers

陈志杰　清华大学

In this talk, I will talk about our recent process about the existence of conformal metric of constant curvature 1 with conic singularities on the sphere. We show that the existence of co-axial metrics implies the algebraic dependence of the singularities. This is based on joint work with Professors Chang-Shou Lin and Yifan Yang.

Periodic solutions of fractional Allen-Cahn equation

杜卓然　湖南大学

In this talk, for fractional Allen-Cahn equation, we will give some qualitative and quantitative results of its periodic solutions, which include the existence, the value of least period, axial symmetry, and the relation about the limit of periodic solutions and the layer solution.

Uniqueness and existence results of mean field equations

胡烨耀　中南大学

We prove a sharp uniqueness result of a mean field equation on arbitrary flat tori. We first establish the evenly symmetry of the solutions by the newly invented sphere covering inequality which is of independent interest. Then we employ a careful nodal line analysis together with a homotopic argument to show the uniqueness. Blow-up solutions are also constructed in both dimension two and four.

Smooth interior approximation of sets and traces of weakly regular vector fields on rough domains

李沁峰　湖南大学

In this talk, I will prove necessary and sufficient conditions for a domain to have strict interior approximation by smooth sets, in the sense that perimeters of the sets also converge to the perimeter of the domain. As a consequence, we are able to extend classical trace theorems on BV functions and bounded divergence measure fields. Potential applications are shape optimization theorems on rough domains. The talk is based on joint work with Guiqiang Chen from University of Oxford, Changfeng Gui from UTSA, Yeyao Hu from CSU, and Monica Torres from Purdue University.

The spectral gap to torsion problem for some non-convex domains

罗鹏　华中师范大学

In this paper, we study the following torsion problem

$$\begin{cases} -\Delta u = 1 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

Let $\Omega \subset \mathbb{R}^2$ be a bounded, convex domain and $u_0(x)$ be the solution of above problem with its maximum $y_0 \in \Omega$. Steinerberger [2018, JFA] proved that there are universal constants $c_1, c_2 > 0$ satisfying

$$\lambda_{\max}\left(D^2 u_0(y_0)\right) \leq -c_1 \exp\left(-c_2 \frac{\text{diam}(\Omega)}{\text{inrad}(\Omega)}\right).$$

And in [Steinerberger, 2018, JFA], he proposed following open problem: "Does above result hold true on domains that are not convex but merely simply connected or perhaps only bounded? The proof uses convexity of the domain Ω in a very essential way and it is not clear to us whether the statement remains valid in other settings." Here by some new idea involving the computations on Green's function, we compute the spectral gap $\lambda_{\max} D^2 u_0(y_0)$ for some non-convex smooth bounded domains, which gives a negative answer to above open problem. This is a jointed work with Hua Chen.

On two phase transition problems and the exact solutions

罗森平　江西师范大学

We shall introduce two phase transition problems proposed by Mueller-Ho (2002, PRL) and Messina-Lowen(2003, PRL) arising in BEC and Wigner bilayers. We give exact solutions to these problems.

Rigidity and gap theorem for Liouville's equation

沈伟明　首都师范大学

In this talk, we study the properties of the first global term in the polyhomogeneous expansions for Liouville's equation. We obtain rigidity and gap results for the boundary integral of the global coefficient. We prove that such a boundary integral is always nonpositive, and is zero if and only if the underlying domain is a disc. More generally, we prove some gap theorems relating such a boundary integral to the number of components of the boundary. The conformal structure plays an essential role. We also give some positive mass theorem type results through the integral of the global coefficient.

Sign-changing solutions for the nonlinear Schrödinger-Poisson system

帅伟　华中师范大学

In this talk, we discuss the existence of sign-changing solution for nonlinear Schrödinger-Poisson system with subcritical or critical growth. The existence and multiplicity of sign-changing solutions for nonlinear Schrödinger-Poisson system with indefinite nonlinearities are also investigated.

Some progress on nonlocal differential inequalities on manifolds

孙玉华　南开大学

We will present some recent results on fractional differential inequality on Riemannian manifolds, the criteria on existence and nonexistence of positive solution are investigated.

Infinitely many nonradial positive solutions for multi-species nonlinear Schrödinger systems in $\mathbb{R}^N$

吴元泽　中国矿业大学

In this talk, I will report our recent results, based on the joint work of Doctor Tuoxing Li and Professor Juncheng Wei, on the multi-species nonlinear Schrödinger systems in $\mathbb{R}^N$:

$$\begin{cases} -\Delta u_j + V_j(x)u_j = \mu_j u_j^3 + \displaystyle\sum_{i=1;i\neq j}^{d} \beta_{i,j} u_i^2 u_j & \text{in } \mathbb{R}^N, \\ u_j(x) > 0 & \text{in } \mathbb{R}^N, \\ u_j(x) \to 0 & \text{as } |x| \to +\infty, \quad j = 1, 2, \cdots, d, \end{cases}$$

Where $N = 2, 3$, $\mu_j > 0$ are constants, $\beta_{i,j} = \beta_{j,i} \neq 0$ are coupling parameters, $d \geq 2$ and $V_j(x)$ are potentials. By Lyapunov-Schmidt reduction arguments, we construct infinitely many nonradial positive solutions of the above system under some mild assumptions on potentials $V_j(x)$ and coupling parameters $\beta_{i,j}$, without any symmetric assumptions on the limit case of the above system. Our result, giving a positive answer to the conjecture of Pistoia and Vaira, and extending their results, reveals a new phenomenon in the case of $N = 2$ and $d = 2$ and is almost optimal for the coupling parameters $\beta_{i,j}$.

Saddle solutions for the fractional Choquard equation

夏健康　西北工业大学

In this talk, I will present some results on the entirely nodal solutions for the fractional Choquard equation. By introducing the Coxeter symmetry, we constructed saddle solutions for this doubly nonlocal equation. These saddle solutions illustrate the nonlocal feature of the Choquard equation and in contrast the local counterpart of Schrödinger equation cannot have such type solutions. This is a joint work with Dr. Yin-Xin Cui in Shanxi Normal University.

Regularity theory for the Dirichlet problem of porous media equations and fast diffusion equations

熊金钢　北京师范大学

It was proved in 1980s that nonnegative solutions of the homogenous Dirichlet problem of porous media equations and fast diffusion equations are Hölder continuous. Due to the loss of uniform parabolicity near the boundary, the higher order regularity was left unsolved. It was asked explicitly by Berryman-Holland 1980 for the fast diffusion equations. In this talk, I will present our recent complete solution to this problem. This is based on a series of work with T. Jin as well as a most recent one with T. Jin and X. Ros-Oton.

The blow-up analysis on $B_2^{(1)}$ affine Toda system: Local mass and affine Weyl group

杨文　中国科学院精密测量科学与技术创新研究院

In this talk, I shall report a result on the affine Toda system with singularity, where the relation between the quantization information from the blowup analysis and the associated algebraic structure for the $B_2^{(1)}$ Toda system is presented. This is a joint work with L.L. Cui, J.C. Wei and L. Zhang.

Uniqueness of single peak solutions for coupled nonlinear Gross-Pitaevskii equations with potentials

曾小雨　武汉理工大学

For a couple of singularly perturbed Gross-Pitaevskii equations, we first prove that the single peak solutions concentrating on the same point are unique provided that the Taylor's expansion of potentials around the concentration point has the same order along all directions. Moreover, for the radially symmetric ring-shaped potential, which attains its minimum at the spheres

$$\Gamma_i := \{x \in \mathbb{R}^N : |x| = A_i > 0\}, i = 1, 2, \cdots, l,$$

and is totally degenerate in the tangential space of Γ_i, we prove that the positive ground state is cylindrically symmetric and is unique up to rotations around the origin.

Group invariant solutions for the planar Schrödinger-Poisson equation

周刚龙　华东师范大学

In this talk, I will introduce the following planar Schrödinger-Poisson system

$$\begin{cases} -\Delta u + V(x)u + \phi u = f(x,u), x \in \mathbb{R}^2, \\ \quad\quad -\Delta \phi = u^2, x \in \mathbb{R}^2. \end{cases}$$

By assuming that $V(x)$, $f(x,u)$ have some certain symmetry for $x \in \mathbb{R}^2$ and the nonlinearity $f(x,u)$ has critical exponential growth with respect to $H^1(\mathbb{R}^2)$, we obtain a nontrivial solution or a ground state solution of Nehari type to the above system. We considered all the cases $p \geq 2$, and we show the existence of solutions with multiple types of symmetry.