The Dirichlet Problem for the Logarithmic Laplacian

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In this talk, we study the logarithmic Laplacian operator L_Δ, which is a singular integral operator with symbol $2\log|\zeta|$. We show that this operator has the integral representation

$$L_\Delta u(x) = c_N \int_{\mathbb{R}^N} \frac{u(x)1_{B_1(x)}(y) - u(y)}{|x-y|^N} dy + \rho_N u(x)$$

with $c_N = \pi^{-\frac{N}{2}}\Gamma(\frac{N}{2})$ and $\rho_N = 2\log 2 + \psi(\frac{N}{2}) - \gamma$, where Γ is the Gamma function, $\psi = \frac{\Gamma'}{\Gamma}$ is the Digamma function and $\gamma = -\Gamma'(1)$ is the Euler Mascheroni constant. This operator arises as formal derivative $\partial_s(-\Delta)^s\Big|_{s=0}$ of fractional Laplacians at $s=0$. We develop the functional analytic framework for Dirichlet problems involving the logarithmic Laplacian on bounded domains and use it to characterize the asymptotics of principal Dirichlet eigenvalues and eigenfunctions of $(-\Delta)^s$ as $s \to 0$.