Growth Rate of Zeros of Almost all Random Entire Functions

Zhuan Ye
Department of Mathematics and Statistics
University of North Carolina University Wilmington

ABSTRACT

Let $f(z,\omega) = \sum_{j=1}^{\infty} A_j(\omega) z^j$ be a random entire function, where $A_j(\omega)$ are independent and identically distributed random variables defined on a probability space $(\Omega, \mathcal{F}, \mu)$. If A_j's are either equal to $e^{2\pi i\theta_j(\omega)}$ with $\theta_j(\omega)$ being of standard uniform distribution or Gaussian random variables with standard Gaussian distribution, then we prove that for almost all $f(z,\omega)$ and any $a \in \mathbb{C}$ with $f(0,\omega) - a \neq 0$, there is r_0 such that when $r > r_0$,

$$\frac{1}{2\pi}\int_0^{2\pi} \log^+ |f(re^{it},\omega)|\, dt = \int_0^r \frac{n(t,\omega,a,f)}{t}\, dt + \log|f(0,\omega) - a| + O(1),$$

where $n(t,\omega,a,f)$ is the number of zeros of $f(z,\omega) - a$ in $|z| < t$ and $O(1)$ is independent of ω. The identity can be regarded as the Nevanlinna's second main theorem and has improved some previous theorems on the growth rate of zeros of random entire functions.