Title.

Higgs Bundles and Hyperbolicities

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Abstract. The theory of period maps has been powerful in the study of higher dimensional Shafarevich program under the assumption of the injectivity of Kodaira-Spencer deformation on Hodge bundles. Viehweg-Zuo constructed a class of Higgs bundles on moduli spaces by combining Kodaira-Spencer deformation and Hodge theory, which is accessible in the most general situation, where period maps fail being locally injective. More explicitly, for a family $f : X \to Y$ parametrizing n-folds F of semi ample ω_F and with the degeneration over a closed subvariety $D \subset Y$, Viehweg-Zuo introduced a Higgs bundle (G, τ) over Y with singularities over D by extending the Kodaira-Spencer deformation τ on the higher order cohomologies of the tangent bundle along F. The central feature of this Higgs bundle is that there exists a natural comparison map $\rho : (G, \tau) \to (E, \theta)$, where (E, θ) is the graded Higgs bundle of the variation of Hodge structures of the relative middle cohomology on the cyclic cover $g : Z_s \to X \to Y$ defined by a section from the linear system of the relative pluri-canonical line bundle on X twisting a small anti ample line bundle on Y. The Hodge metric on ker(θ) becomes a non-zero (possibly degenerated) negatively curved Finsler metric on $Y \setminus D =: U$ via the iterated Kodaira-Spencer deformation and if the second graded piece $\rho^{n-1,1}$ of ρ is injective on T_U. This Finsler metric plays a crucial roll in the study of hyperbolicities of U by many people. Indeed $\rho^{n-1,1}$ holds being injective for two exteme cases $\kappa(F) = n$ and $\kappa(F) = 0$ by Bogomolov vanishing theorem and the trivial reason. For the general case $0 \leq \kappa(F) \leq n$ Viehweg-Zuo showed that it is generically injective along any algebraic curve in U. Very recently X.Lu, R.R. Sun and K. Zuo showed $\rho^{n-1,1}_{T_U}$ is injective for $\kappa(F) = 1$ by investigating Iitaka fibration. Besides Brody, Kobayashi and Viehweg hyperbolicities we rise a conjecture on Borel and topologic hyperbolicities. If time permits I shall outline an approach towards to the conjecture.

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