

# **Workshop and Summer School on Lie Theory and Representation Theory**

## **Program**



**July 10 - July 30, 2006**

**Department of Mathematics,  
East China Normal University  
Shanghai, China**

# **Workshop and Summer School on Lie Theory and Representation Theory**

July 10–July 30, 2006

(<http://math.ecnu.edu.cn/xs/li/li.htm>)

## **Sponsored by:**

East China Normal University (ECNU) and  
Department of Mathematics, ECNU

## **Organized by:** Department of Mathematics, ECNU and

Naihong Hu

Lei Lin

Jiabao Lu,

Hebing Rui

Jianyi Shi

Bin Shu (Coordinator)

Jianpan Wang

**Place (Workshop July 10–July 12):** Hall 504, Science Building A and

**(Summer School July 13–July 30):** Classroom 211, Wenshi Building

## **Time Table of Workshop**

| Time        | July 10 (Mon)  | July 11 (Tue) | July 12 (Wed)  |
|-------------|--|---------------|----------------|
| 8:30–9:30   | Registration<br>(8:30–9:30) and<br>Welcome Speeches<br>(9:30–9:45) | N. Xi         | A. Premet      |
| 9:45–10:45  | T. Tanisaki  | S. Doty       | R. Farnsteiner |
| 11:15–12:15 | Z. Lin   | Y. Gao        | W. Wang        |
| Lunch Time  |  |               |                |
| 13:45–14:45 | K. Zhao  | S. Tan        | Y. Lin         |
| 15:00–16:00 | H. Zhang   | P. Zhang      | J. Hu          |
| 16:30–17:30 | Y. Zhang   | F. Li         | Y. Su          |

# Program of Workshop

## July 10 (Mon)

- 9: 35 - 9:45 Welcome Speeches by Professors Jianpan Wang and Feng Zhou  
9: 45– 10:45 T. Tanisaki (Osaka City University)  
On quantized flag manifolds  
10:45-11:15 Tea Time  
11:15— 12:15 Zongzhu Lin (Kansas State University)  
TBA

### Lunch Time

- 13: 45— 14:45 Kaiming Zhao (Academia Sinica & Wilfrid Laurier University)  
Representations of the twisted Heisenberg-Virasoro algebras  
15: 00 - 16:00 Hechun Zhang (Tisinghua University)  
Canonical bases of tensor modules of quantum supergroup  
16:00-16:30 **Tea Time**  
16:30 - 17:30 Yongzheng Zhang (Northeast Normal University)  
Some researches on modular Lie superalgebras

## July 11 (Tue)

- 8: 30 - 9:30 Nanhua Xi (Academia Sinica)  
Representations of Affine Hecke Algebra of Type  $G_2$   
9: 45– 10:45 S. Doty (Loyola University)  
Generators and relations for generalized  $q$ -Schur algebras  
10:45 - 11:15 **Tea Time**  
11:15— 12:15 Yun Gao (York University)  
Spin representations and BC-graded Lie algebras

### Lunch Time

- 13: 45— 14:45 Shaobin Tan (Xiaman University)  
TBA  
15: 00— 16: 00 Pu Zhang (Shanghai Jiaotong University)  
Serre dualities and Calabi-Yau Categories  
16:00-16:30 **Tea Time**  
16:30 — 17:30 Fang Li (Zhejiang University)  
TBA  
17:45 - **Group Photo**  
18:30 - **Banquet**

## July 12 (Wed)

- 8: 30 - 9:30 A. Premet (The University of Manchester)  
Symmetric invariants of centralizers in reductive Lie algebras
- 9: 45– 10:45 R. Farnsteiner (University of Bielefeld )  
Affine Quivers, Polyhedral Groups, and Representation Type
- 10:45-11:15 **Tea Time**
- 11:15– 12:15 Weiqiang Wang (University of Virginia)  
Spin Hecke algebras

### Lunch Time

- 13: 45– 14:45 Yanan Lin (Xiaman University)  
TBA
- 15: 00– 16: 00 Jun Hu (Beijing Institute of Technology)  
Mullineux involution and twisted affine Lie algebras
- 16:00-16:30 **Tea Time**
- 16:30 – 17:30 Yucai Su (The University of Science and Technology of China)  
TBA

## Abstracts of Workshop Talks

✧ Fang Li (Zhejiang University)

**Characterization of Left Artinian Algebras Through Pseudo Path Algebras** (July 11, 16:30-17:30) : In this talk, using pseudo path algebras, we generalize Gabriel's Theorem on elementary algebras to left Artinian algebras over a field  $k$  when the quotient algebra can be lifted, in particular, when the dimension of the quotient algebra decided by the  $n$ 'th Hochschild cohomology is less than  $2$  (for example,  $k$  is finite or  $\text{char} k=0$ ). Using generalized path algebras, the generalized Gabriel's Theorem is given for finite dimensional algebras with 2-nilpotent radicals. As a tool, the so-called pseudo path algebras are introduced as a new generalization of path algebras, which can cover generalized path algebras (see Fact 2.5). The main result is that (i) for a left Artinian  $k$ -algebra  $A$  and  $r=r(A)$  the radical of  $A$ , when the quotient algebra  $A/r$  can be lifted, it holds that  $A \cong \text{PSE}_k(\Delta, \mathcal{A}, \rho)$  with  $J^s \subset \langle \rho \rangle \subset J$  for some  $s$  (Theorem 3.2); (ii) for a finite dimensional  $k$ -algebra  $A$  with  $r=r(A)$  2-nilpotent radical, it holds that  $A \cong k(\Delta, \mathcal{A}, \rho)$  with  $J^2 \subset \langle \rho \rangle \subset \widetilde{J^2} + \widetilde{J} \cap \text{Ker} \varphi$  (Theorem 4.3), where  $\Delta$  is the quiver of  $A$

and  $\rho$  is a set of relations. For all cases we discuss in this paper, the uniqueness of such quivers  $\Delta$  and generalized path algebra/pseudo path algebras satisfying the isomorphism relations is obtained (see Theorem 3.5 and 4.4).

※ **Shaobin Tan** (Xiaomen University)

**Lie algebras related to  $A_1$ -EALA** (July 11, 13:45-14:45) : In this talk we first recall the notion of extended affine Lie algebra, or EALA for short. Especially we will talk the structures of the EALAs of type  $A_1$ . Then we introduce a class of Lie algebras based on a commutative ring and  $\mathfrak{so}_3$ , which contains the usual  $A_1$  toroidal Lie algebras as special cases, and in general this class of Lie algebras is nongraded. We will deal the classification, central extensions, derivations and vertex operator representation for this kind of Lie algebras.

※ **Toshiyuki Tanisaki** (Osaka City University)

**On quantized flag manifolds** (July 10, 9:45-10:45): I will talk about some topics on the quantized flag manifolds as non-commutative schemes. I would also like to present some problems concerning the quantized flag manifolds at roots of unity.

※ **Pu Zhang** (Shanghai Jiaotong University)

**Serre dualities and Calabi-Yau Categories** (July 11, 15:00-16:00): In this talk I will explain how the representation theory of algebras relates the Serre dualities and the Calabi-Yau categories; I'll give an equivalent definition of a Calabi-Yau category; and show some constructions of Serre dualities and Calabi-Yau categories, in particular, I'll justify the singularity category (as a quotient triangular category of the bounded derived category, which is induced by the tilting modules) has a Serre duality.



# Program of Summer School

## Lecturers

- L= Zongzhu Lin (Kansas State University)  
 F= Rolf Farnsteiner (University of Bielefeld)  
 P= Alexander Premet (The University of Manchester)  
 T= Toshiyuki Tanisaki (Osaka City University)  
 W= Weiqiang Wang (University of Virginia)

## Time Table of Summer School

| Jul. 13<br>-Jul. 28 | 13<br>Thu. | 14<br>Fri. | 15<br>Sat. | 17<br>Mon. | 18<br>Tue. | 19<br>Wed. | 20<br>Thu. | 21<br>Fri. | 24<br>Mon. | 25<br>Tue. | 26<br>Wed. | 27<br>Thu. | 28<br>Fri. |
|---------------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 9:30-<br>10:30      | P          | P          | F          | P          | P          | P          | P          | W          | L          | W          | L          | L          | L          |
| 11:00-<br>12:00     | T          | T          | F          | T          | T          | T          | P          | W          | L          | W          | L          | L          | L          |
| Lunch<br>time       |            |            |            |            |            |            |            |            |            |            |            |            |            |
| 14:00-<br>15:00     | F          | F          |            | F          | F          | F          | W          | L          | W          | L          |            |            |            |
| 15:30-<br>16:30     | F          | F          |            | F          | F          | F          | W          | L          | W          |            |            |            |            |

| July 29-July 30 | 29 Sat | 30 Sun. |
|-----------------|--------|---------|
| 9:30-10:30      | L      | L       |
| 11:00-12:00     | L      | L       |

※ July 15 afternoon and July 16 all day, we are planning an excursion to Hangzhou.

※ July 22 - July 23, we are planning various excursions around Shanghai city.

To see Information Page for more details concerning the excursions.

## **Contents of Summer School Lectures**

### **Combinatorial and Geometric Aspects of the Representation theory of Finite algebraic groups**

#### **-----Rolf Farnsteiner's Lectures (12 one-hour lectures)**

1. Basic Properties of Hopf Algebras
2. Group Schemes
3. Algebras of Measures
4. Restricted Lie algebras
5. The Heller Operator
6. Complexity
7. Representation Type
8. The Friedlander-Suslin Theorem
9. The Space of p-Points
10. Tameness and Complexity
11. Blocks of Frobenius Kernels of Smooth Groups
12. Algebraic Families of Vector Spaces
13. Schemes of Tori
14. Gabriel's Theorem
15. Hereditary Algebras
16. A Criterion for Wildness
17. Trivial Extensions
18. McKay Quivers
19. Nagata's Theorem
20. Tensor Products of Simple and Principal Indecomposable Modules
21. Finite Representation Type
22. Binary Polyhedral Groups
23. Tame Representation Type

#### **Prerequisites**

- \_ standard facts from homological algebra,
- \_ basic Lie theory,
- \_ a little algebraic geometry and commutative algebra.

#### **Zongzhu Lin's Lectures (15 one-hour lectures)**

I will give 5-6 lectures during 19-25 (one lecture per day, or 2 lectures per day for three days, is already a lot given there are four or three other lectures). I will give 9-10 lectures for the following week (if I am the only lecturer) with a two-hour lecture every day.

Lecture 1. Hall algebras over finite fields, greens theorems on multiplications and comultiplications, Hopf pairings, and twisted multiplications.

Lecture 2. Composition algebras and relations to quantum groups and Drinfeld double construction and generic subalgebras.

Lecture 3. Geometric Approach to Hall algebras and composition algebras, canonical bases.

Lecture 4. Describing the canonical bases for affine quivers using the Auslander-Reiten quivers of the representations of the affine quivers.

Lecture 5. Quivers with loops, quantized enveloping algebras of generalized Kac-Moody Lie algebras.

(I can reorganize the lectures into three 2-hour lectures and include one comparing the support varieties of Lie algebras and finite Chevalley groups, or this topic could be talked at the workshop, everything depends on your scheduling)

In the 9-10 lectures after 25th, I will talk on  $l$ -adic cohomology and perverse sheaves. They will include derived categories, triangulated categories,  $t$ -structures and heart of  $t$ -structures; Locally constant sheaves, Borel-Moore cohomologies and cohomologies with compact supports. Etale morphisms and Etale topology on schemes and  $l$ -adic sheaves and  $l$ -cohomology. The category of perverse sheaves the properties. Depends on students interests, I might include Springer representations of the Weyl groups and relations to nilpotent varieties.

### **Alexander Premet's Lectures (7 one-hour lectures)**

I plan to start with the classification of nilpotent orbits in good characteristic. I will present this in detail by using my recent approach via the Kempf-Rousseau theory. Next I will discuss the support varieties of blocks in nonrestricted module categories. I'll discuss recent results of Bezrukavnikov, Mirkovic and Rumynin on Lusztig's conjecture which relates simple modules in blocks of reduced enveloping algebras with Borel-Moore homology of Springer fibers. I'll outline a deformation-theoretic proof of the Kac-Weisfeiler conjecture obtained jointly with Skryabin and discuss a possible generalization of this proof to other classes of Lie algebras. After that I plan to introduce the generalized Gelfand-Graev modules for reduced enveloping algebras. The endomorphism algebras of these projective modules are finite dimensional versions of the so-called finite  $W$ -algebras of mathematical physics. I'll give some examples of finite  $W$ -algebras, and will treat in detail the case of the minimal nilpotent orbit, both in characteristic 0 and characteristic  $p$ . A link with the Joseph ideal and Goldie rank polynomials will be established.

**Introduction to D-modules and their connection with representation theory.**  
**-----T. Tanisaki's Lectures (5 one-hour lectures)**

Abstract: I will give a survey on the theory of D-modules and its application to the representation theory. My plan is as follows.

Lecture 1-3 Introduction to D-modules Sato's philosophy, holonomic systems, Riemann-Hilbert correspondence

Lecture 4,5 Application to the representation theory Beilinson-Bernstein correspondence, character formulas of Kazhdan-Lusztig type

**Introduction to Lie superalgebras**  
**-----Weiqiang Wang's Lectures (8 one-hour lectures )**

Abstract: The field will always be assumed to be zero characteristic.

I will review quickly the basics of simple Lie algebras and their representations, with emphasis on type A case. We introduce the Schur duality and Howe's reductive dual pairs.

I will introduce the notion of Lie superalgebras, and review Kac's classification of finite-dimensional simple Lie superalgebras. We will put emphasis on the type A and type Q. I will explain an analog of the Schur and Howe duality in these cases.

I will then explain the central characters and blocks of representations for Lie superalgebras. Then we will address the irreducible character problems in the type A (and type Q if time permitting) using a version of Kazhdan-Lusztig theory developed by Serganova and Brundan.



# List of Participants

|                     |  |                 |
|---------------------|--|-----------------|
| Stephen Doty,       | Loyola University,                                     | Chicago         |
| Rolf Farnsteiner,   | University of Bielefeld,                               | Bielefeld       |
| Yun Gao,            | York University,                                       | Toronto         |
| Jun Hu,             | Beijing Institute of Technology,                       | Beijing         |
| Fang Li,            | Zhejiang University,                                   | Hangzhou        |
| Weiqliang Lin,      | The University of Science and Technology of China,     | Hefei           |
| Yanan Lin,          | Xiamen University,                                     | Xiamen          |
| Zongzhu Lin,        | Kansas State University,                               | Manhattan       |
| Dong Liu,           | Shanghai Jiaotong University,                          | Shanghai        |
| Wende Liu,          | Harbin Normal University,                              | Harbin          |
| Daoji Meng,         | Nankai University,                                     | Tianjin         |
| Alexander Premet,   | The University of Manchester,                          | Manchester      |
| Yanming Pu,         | Tongji University,                                     | Shanghai        |
| Phill Schultz,      | The Univ. West. Australia,                             | Crawley         |
| Guangai Song,       | Shandong Institute of Business,                        | Shandong        |
| Yucai Su,           | The University of Science and Technology of China,     | Hefei           |
| Shaobin Tan,        | Xiamen University,                                     | Xiamen          |
| Toshiyuki Tanisaki, | Osaka City University,                                 | Osaka           |
| Weiqliang Wang,     | University of Virginia,                                | Charlottesville |
| Xiandong Wang,      | Qingdao University,                                    | Qingdao         |
| Zixiang Wang,       | Wuhan University,                                      | Wuhan           |
| Zixiang Wu,         | Zhejiang University,                                   | Hangzhou        |
| Nanhua Xi,          | Academia Sinica,                                       | Beijing         |
| Limeng Xiao,        | Hebei University,                                      | Baoding         |
| Jiachen Ye,         | Tongji University,                                     | Shanghai        |
| Hechun Zhang,       | Tsinghua University,                                   | Beijing         |
| Pu Zhang,           | Shanghai Jiaotong University,                          | Shanghai        |
| Xiufu Zhang,        | Xuzhou Normal University,                              | Xuzhou          |
| Yongzheng Zhang,    | Northeastern China Normal University,                  | Changchun       |
| Kaiming Zhao,       | Academia Sinica, Beijing & Wilfrid Laurier University, | Waterloo        |
| Lei Zhao,           | University of Virginia,                                | Charlottesville |
| Linsheng Zhu,       | Changshu Institute of Technology,                      | Changshu        |
| 周忠国                 | 同济大学   |                 |
| 佟洁                  | 同济大学   |                 |
| 胡建华                 | 同济大学   |                 |
| 王惟嘉,                | 同济大学   |                 |
| 李静,                 | 同济大学   |                 |

|                 |             |
|-----------------|-------------|
| 张雁磊,            | 同济大学        |
| 单翠萍.            | 同济大学        |
| 吴隋超             | 同济大学        |
| 李军波             | 上海交通大学      |
| 程永胜             | 上海交通大学      |
| 姜 伟             | 上海交通大学      |
| 吴月柱             | 上海交通大学      |
| 张成林             | 上海交通大学      |
| 李志伟             | 上海交通大学      |
| 杨恒云             | 上海交通大学      |
| 高寿兰             | 上海交通大学      |
| 申 冉             | 上海交通大学      |
| 蒋启芬             | 上海交通大学      |
| 高 楠             | 上海交通大学      |
| 岳小青             | 上海交通大学      |
| 付佳瑗             | 上海交通大学      |
| 程东明             | 浙江大学        |
| 田红禹             | 浙江大学        |
| 黄成宝             | 浙江大学        |
| 卢燕骅             | 浙江大学        |
| 魏 竹             | 东北师范大学      |
| 洪久族             | 中国科学院       |
| 陈洪佳             | 中国科技大学      |
| 尚士魁             | 中国科技大学      |
| 陈小伍             | 中国科技大学      |
| 吴伊涛             | 中国科技大学      |
| 姚 远             | 中国科技大学      |
| 赵 青             | 中国科技大学      |
| 方 明             | 中国科技大学      |
| 常智华             | 中国科技大学      |
| 孙建才             | 青岛大学/常熟理工学院 |
| 王 琦             | 西华师大        |
| 吴明忠             | 西华师大        |
| 杜婉娟             | 西华师大        |
| 丁 明 (7.20-7.30) | 清华大学        |
| 李 彬 (7.20-7.30) | 清华大学        |
| 法焕霞             | 常熟理工学院      |
| 曾 波             | 厦门大学        |
| 连海峰             | 厦门大学        |
| 王 清             | 厦门大学        |
| 孙晓丽             | 厦门大学        |
| 余银娜             | 厦门大学        |

|       |        |
|-------|--------|
| 郑兆娟   | 厦门大学   |
| 李小朝   | 信阳师范学院 |
| 别 潇   | 信阳师范学院 |
| 王改霞   | 信阳师范学院 |
| 王宪栋学生 | 青岛大学   |
| 顾颐臣   | 苏州大学   |
| 王松    | 苏州大学   |
| 徐杰    | 华东师大   |
| 司梅    | 华东师大   |
| 郭琼    | 华东师大   |
| 连晓丹   | 华东师大   |
| 徐善顶   | 华东师大   |
| 杨晓薇   | 华东师大   |
| 刘畅    | 华东师大   |
| 李宜阳   | 华东师大   |
| 贺谊浩   | 华东师大   |
| 姚裕丰   | 华东师大   |
| 张晓东   | 华东师大   |
| 常蓝月   | 华东师大   |
| 王小双   | 华东师大   |
| 郑立笋   | 华东师大   |
| 彭帆    | 华东师大   |
| 左怀青   | 华东师大   |
| 段永健   | 华东师大   |
| 王卫国   | 华东师大   |
| 白晓棠   | 华东师大   |
| 王秀玲   | 华东师大   |
| 张红莲   | 华东师大   |
| 裴玉峰   | 华东师大   |
| 陈如    | 华东师大   |
| 张姣    | 华东师大   |
| 张美蓉   | 华东师大   |
| 张云    | 华东师大   |
| 谢长庚   | 华东师大   |
| 易国强   | 华东师大   |

# Information

**Email Rooms:** A 1403, Science Building / Open time: 8:00-18:00, 19:00-21:00

**Common Office for Lecturers:** A 1405, Science Building

**Group Photos for Workshop:** We will take a group photo of all participants in front of Science Building after talks in the afternoon of Tuesday (July 11) or during the teatime of Wednesday (July 12) morning. It's up to the weather.

**Banquet:** There will be a banquet on July 11 (Tuesday) from 6:00 in the dinning hall of Yi Fu Hotel. The invited people to the banquet are limited to be the speakers and the university teacher participants.

**Cafeteria:** Hedong Cafeteria (neighbouring Wenshi Building, the lecture place of Summer School), Hexi Cafeteria, Liwa Cafeteria, the restaurant of Yifu Hotel and the restaurant of Service Center of International Exchange.

**Excursions and entertainments:** We are planning an excursion to Hangzhou for one-day and half. Hangzhou is one of the most famous sightseeing city in China, which lies southeast off Shanghai, with two-hour train trip from Shanghai. In principle, all participants of Summer School are welcome to participate in such an excursion, with fare around 390 RMB (covering all board and lodging during the excursing).

We will also plan some other entertainment and sightseeing during the summer school leisure time, according to the common interest of participants. The options include enjoying Huang Pu River Cruise at night, visiting Shanghai History Museum, watching Beijing Opera or Yue Opera, touring the old town of Shanghai and touring the modern town of Shanghai in Pudong (visiting Jinmao Tower or Oriental Pearl Tower).

The other recommendable tours around Shanghai are: to Zhouzhuang, Zhujiajiao, Xitang and Sheshan.

For more entertainment and tourism information, you can get from the following webpage <http://www.meet-in-shanghai.net/default.asp>