The Cuntz semigroup as an invariant for C*-algebras

Toms has constructed examples of simple C*-algebras (based on examples of Villadsen) which can only be distinguished by means of the Cuntz semigroup. This suggests that one should pursue the study of this invariant, in the hope that, eventually, one might be able to use it to prove isomorphism results.

To begin with, it would seem important to try to compute this invariant in the simplest cases (not those referred to above!). Considerable progress has been achieved in this direction. The result is that all known classification results for simple (separable, amenable) C*-algebras can be expressed in terms of the Cuntz semigroup, together with the abelian group $K_1$. Interestingly, the Cuntz semigroup has been shown to be a complete invariant also for certain non-simple C*-algebras, namely, inductive limits of matrix algebras over (second countable) locally compact spaces of dimension one—provided that the $K_1$-group is zero. (This is joint work with Ciuperca.)

For interesting examples of such algebras there are no non-zero simple subquotients. In this case, the Cuntz semigroup can also be computed, in terms of the cone of lower semicontinuous traces (not necessarily densely defined). (This is joint work with Robert and Santiago.)